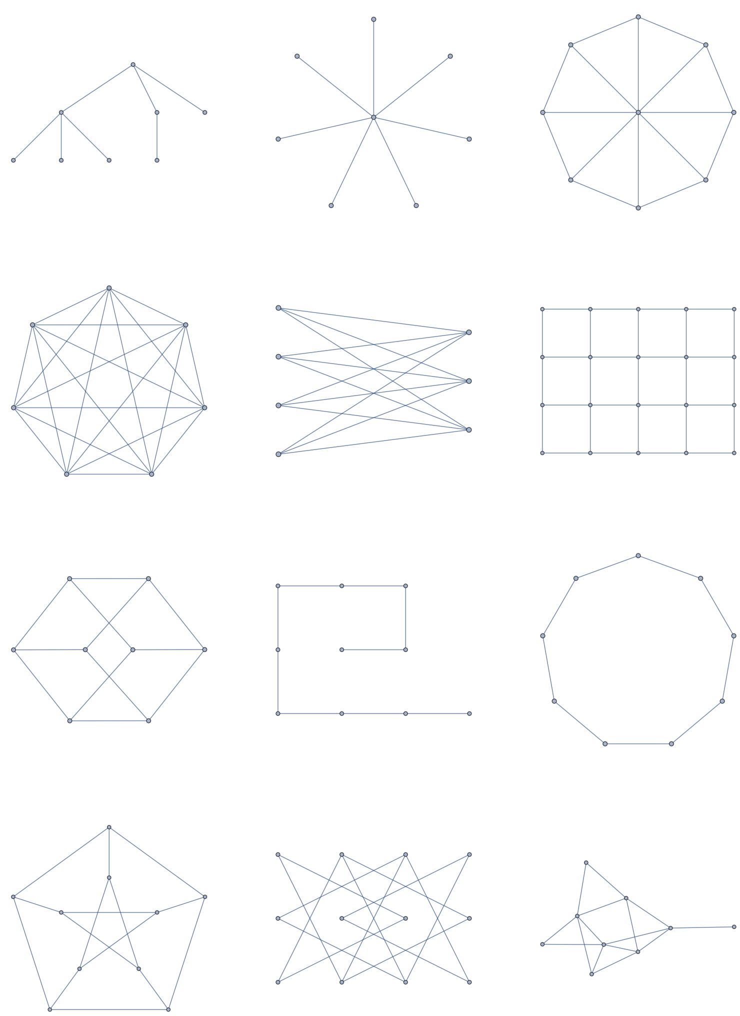
Problem 1. Gandalf is trying to hide two Rings of Power. He wants to make sure they are as far apart from each other as possible, so Sauron can't find them easily. For each of these maps, where should Gandalf put the rings? In each case, what is the distance between the rings?

(This distance is called the *diameter* of a graph.)



Problem 2. Sauron is trying to find the Rings of Power. He wants to build his fortress somewhere so that no other place is too far away. For each of the maps above, where should he build his fortress to minimise the distance to the furthest other point, and what is this distance?

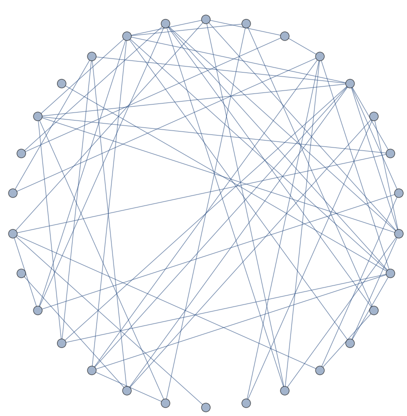
(The distance in this case is called the *radius* of the graph.)

Problem 3. Can you design graphs with the following measurements:

- Diameter 4, radius 3?
- Diameter 7, radius 7?
- Diameter 8, radius 3?
- Diameter 20, radius 10?
- Diameter 5, radius 7?

If there are no graphs with these measurements, explain why.

Problem 4. What are the radius and diameter of this graph:



Trial-and-error might take a while! Can you design a more efficient method to calculate it instead?

Problem 5. In problems 1-4 you saw two ways to measure the "complexity" of a graph: the radius and the diameter. The larger the radius or diameter, the "more complicated" the graph is. Here are a couple other ways to measure complexity:

- **Connected Components:** A connected component of a graph is a set of vertices where any two have a path between them. For example, when a graph has no edges, every vertex is a connected component! One way to measure complexity is by counting the number of connected components.
- **Average degree:** One can count the number of edges leaving each vertex, then take the average of these numbers over all vertices. This is called the average degree of the graph.
- **Number of cycles:** A cycle in a graph is a path that starts and ends at the same vertex without crossing itself. Counting the number of cycles gives another measure of complexity.

Now we have five ways to measure the complexity of a graph. For each of the $\binom{5}{2} = 10$ pairs of measures, find a graph that has different complexities for each, and whose complexity is at least two with respect to each measure. Can you always find a graph which is more complex under either of the measures?

Problem 6. Bonus: What other ways can you think of to measure graph complexity?