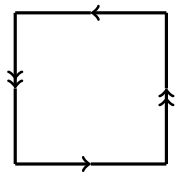
UW Math Circle

February 2, 2017

1. Consider the surface below:



   (a) How many sides does this surface have?

   (b) What surface do you get when you cut this surface down the middle?

   (c) Now cut out a rectangle from the middle of this surface. What surface(s) do you end up with?

2. Take a strip of paper, put a few twists in it, and tape the ends together. How many sides does the resulting surface have? Try a few different times with different number of twists.

3. Take a strip of paper, put one twist in it. This is called a möbius band.

   (a) How many sides does this surface have?

   (b) Draw a presentation of this surface, and use your presentation to figure out what will happen when you cut your surface down the middle.

   (c) Now actually cut your surface down the middle. Is this what you expected? Explain what happens.

4. Use a presentation of a möbius band to figure out what will happen when instead of cutting it down the middle, you cut it down a third of the way from the top. Now make a möbius band, and cut it a third of the way down. Explain what happens.

5. When you have a surface with one edge, like a möbius band, the edge looks like a string with its ends joined together. Draw just the edge for a möbius band with one twists and a möbius band with three twists. Can you transform one into the other without tearing the string or passing it through itself?