UW Math Circle

November 3, 2016

Homework

In today's worksheet, we talked about cups in a line that were facing up or down. Now, our cups can be **facing up, facing down, or lying on their side**.

1. You have **four** cups in a row.

   (a) If the cups aren't labeled, how many ways are there to arrange the cups (they can be up, down, or on their side)?

   (b) If they aren't labeled, how many ways are there to arrange them so exactly two cups are on their side?

   (c) If they aren't labeled, how many ways are there to arrange them so at most two cups are on their side?

   (d) If the cups are labeled $1, 2, 3$ and 4, how many ways are there to arrange them so that exactly three cups are facing down?

2. Now, you have **five** cups in a row, all facing up.

   (a) If a cup is facing up, you are allowed to turn it face down OR put it on its side. If it is facing down, you are allowed to turn it face up OR put it on its side. Finally, if it is on its side, you are allowed to put it face up OR face down. If you are allowed to change the state of **two cups** at a time, can you make all of the cups face down?

   (b) Now, if a cup if facing up, you are only allowed to put it on its side. If it is on its side, you are only allowed to put it face down. If it is face down, you are only allowed to put it face up. Starting with five cups all facing up, if you can only change **two cups** at a time, is it possible to make all of the cups face down?

   (c) In (a) or (b), if it *is possible* to get all of the cups facing down, what is the minimal number of moves it takes to get all of the cups facing down?