A *deterministic game* is one which satisfies the following three properties:

- **No hidden information.** There are no cards which are secret to some players, no surprises that will be revealed as the game progresses, and no screens behind which players hide their unit instructions or battleship configurations.

- **No random chance.** There are no dice rolled, no cards drawn, no tiles pulled, and no lots cast.

- **No ties.** The game should end in a finite amount of time, with one winner. (Although if you'd like to analyze a game which meets the other criteria but has ties, it's easy to fix this by introducing a tiebreaker mechanic.)

We are particularly interested in *two-player* deterministic games, in which players alternate turns making *moves*. If the moves and goals for one player are always identical to those of the other player, then the game is *impartial*. Otherwise (for example, if the player pieces are differentiated by color or if the players have different objectives), the game is *partisan*.

Normal Games

A deterministic game follows the *normal play convention* if the players alternate making moves until one player is stuck and has no possible move. Then, the other player wins.

Here are some **normal impartial deterministic games**.

1. **The Twenty-One Game**: Place 21 stones on the table. (Or, alternatively, draw 21 circles on a piece of paper, or just say the number "21".) On a turn, a player removes either one, two, or three stones. The player who takes the last stone wins (because then the other player can't make a move).
   Variants: This is part of a broader class of games called **Subtraction Games**. Start with other numbers, or allow more stones to be taken away at once, or use entirely different sets of possible moves. What if the number of stones removed must be a power of two? What if it must be a prime number?

2. **Kayles**: Place some number of stones in a line, perhaps with gaps at various places in the line. On a turn, a player removes one stone (anywhere) or two stones that were touching. Whoever takes the last stone wins.
   Variants: This is a **Taking-and-Breaking Game**. Allow more stones to be removed at once, and/or change the rules on when adjacent stones may be removed. For a famously unsolved game, allow only *one* stone to be removed per turn, but require that it touch at least one other stone.

3. **Nim**: Place some number of stones on the table, divided into several piles. On a turn, a player removes *any* number of stones (but not zero), provided that all removed stones came from the same pile. Whoever takes the last stone wins.
   Variants: Allow a player to remove zero stones, and instead split one pile into two smaller piles. Or, combine the rules of Nim with a subtraction game. (For example, create several piles of stones, then allow a player to remove one, two, or three stones from just a single pile.)

4. **Hackenbush**: Start by drawing a *rooted graph*, a network of points and line segments which is connected to a *ground*, as shown below. On a turn, a player erases any one line segment, and then erases anything which is no longer connected to the ground. The player who clears the board wins.
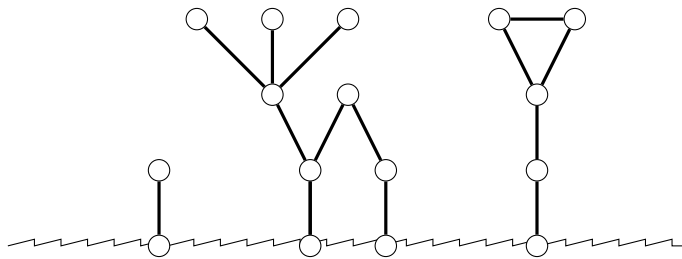
5. **Chomp**: Bake a pan of brownies and cut it into a grid of squares. (Or, y'know, draw it.) Remove the lower-left brownie and set it aside. Players take turns eating any brownie in the pan, along with all other brownies that are above or to the right of it. Whoever eats the last brownie in the pan wins. The first few moves of an example game are shown below, where the gray dot indicates which brownie is being chosen.
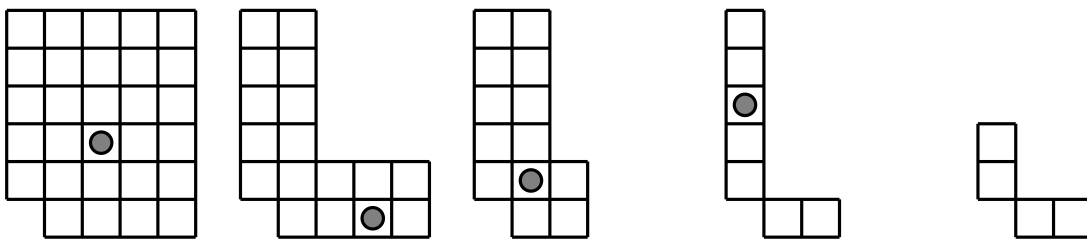
6. **Domineering**: Begin with a chessboard of arbitrary dimensions. On a turn, lay a domino across two adjacent unoccupied squares. A player who cannot fit another domino on the board loses. A few turns are shown below.
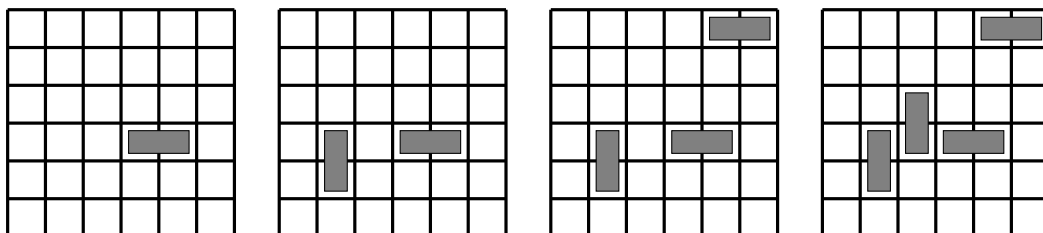
7. **Col**: Begin with an arbitrary map of states, countries, districts, whatever, in which only the outlines of the regions are shown. On a turn, pick an unfilled region and color it pink, yellow, or teal. You may never fill one region with the same color as any region which borders it. A player who can make no legal move is the loser.

Here are some **normal partisan deterministic games**.

1. **Red-Blue Hackenbush**: Draw a rooted graph as in Hackenbush, but color each edge red or blue. One player may only erase blue edges, while the other may only erase red edges. (As usual, any edge which is no longer connected to the ground is removed at the end of each turn, regardless of color.) A player who cannot make a move loses.
   <u>Variant:</u> In **Red-Blue-Green Hackenbush**, a third neutral color may be erased by either player.

2. **Partisan Domineering**: Like impartial domineering, except one player may only lay dominos vertically, while the other may only lay dominos horizontally.

3. **Partisan Col**: Like Col, except one player may only use yellow, while the other may only use pink.
   <u>Variant:</u> In **Snort**, the rules are reversed: a region may never be filled with a *different* color than that of a bordering region.

4. **Cutcake**: Begin with a rectangular cake, subdivided into squares via grid lines of frosting. On a turn, pick a piece of the cake and cut it along a grid line. Once two pieces are separated, move them apart so that a single slice never cuts through more than one piece. One player only makes vertical cuts, while the other only makes horizontal cuts. Whoever cannot make a cut on his or her turn (because their are no remaining grid lines in the appropriate direction) is the winner.
   <u>Variants:</u> In a variant called **Hotcake**, after cutting a piece in two, choose *one* of the resulting smaller pieces and turn it 90° clockwise.


Other Play Rules

Instead of using the normal play rule, any of the above games may instead be played with the **misère rule**. This time, the player who is stuck and cannot make any move is the *winner*. In some cases, the difference this makes is only minor, while in others it considerably alters the underlying strategy.

You might also play **multiple games at once**. Pick any two (or more!) of the previous games, or set up multiple copies of one game side by side. On a turn, select any of the games in progress, and make a move in that game. Play until one player cannot make a move in *any* of the games, at which point that player loses.

There are many popular deterministic games which don't follow any of the normal play rules, but which still yield to similar analytic strategies. (Some of these games allow ties. Choose an arbitrary tiebreaker in advance, such as allowing the first player to win.)

1. **Dots and Boxes**: Begin with a rectangular grid of dots. On a turn, connect two adjacent dots with a line segment. If you've just completed the fourth side of a 1×1 box, put your initial in the box and take another turn. When the grid is completely full, whichever player has initialed the most boxes wins.

2. **The Shannon Switching Game**: Begin with two staggered overlaid grids of gray and white dots, as shown below. On your turn, connect two adjacent dots in your color with a straight line, provided that the line you draw does not cross a line previously drawn by the other player. The gray player wins by creating a network that links a gray dot on the left to a gray dot on the right. The white player wins by creating a network that links a white dot on the top to a white dot on the bottom.
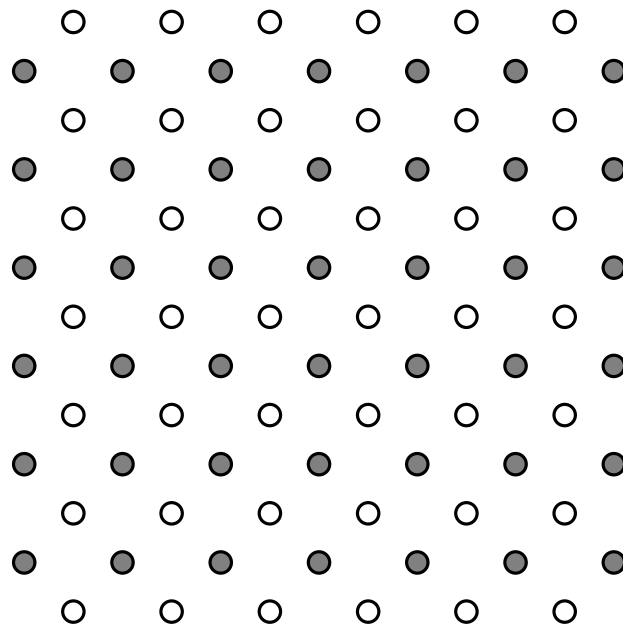
3. **Blokus**: In this commercial board game, each player begins with an identical collection of pieces in various shapes. Players take turns laying pieces in the unoccupied spaces of a large board. Each player's piece after the first must share a corner with one of his or her previously laid pieces, but no two pieces laid by the same player may share a border along an edge. A player who may not fit a piece onto the board simply passes. Once all players pass, whichever player has laid the largest total area of pieces wins. Search for "Blokus" if you'd like more information.