

Halloween Math Auction

UW Math Circle

31 October 2013

Each problem is worth \$100.

1. A *spooky* word is defined to be one with all of its letters in alphabetical order or reverse alphabetical order. For example, “ghost” and “spook” are spooky words. Find as many spooky words with at least 6 letters as you can. For reference, here is the Latin alphabet:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z.

A team has a better solution to this problem if it is able to present a word that the previous team failed to find.

2. A Gandalf is a new chess piece. Once a Gandalf is placed on the board, it does not move, and no piece can move past it (“You shall not pass!”). What is the minimum number of Gandalfs that you can place on an 8×8 chessboard so that it is possible to place 12 queens on the board that do not attack each other?

A team has a better solution to this problem if it is able to present a solution using fewer Gandalfs than the previous team.



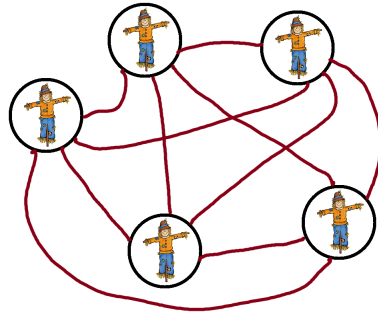
3. Write the number 200 as the sum of a maximal number of distinct prime numbers. For example, this would not be a very good solution (using 2 numbers):

$$200 = 97 + 103.$$

A team has a better solution to this problem if it is able to present a way to do this using more primes than the previous team.

4. Design a corn maze with seven scarecrows, where every pair of scarecrows is connected by a path, with as few as possible crossings of paths. Shown below is a design with five scarecrows and three crossings.

A team has a better solution to this problem if it is able to present a design with fewer crossings than the previous team's design.



5. Four witches, conveniently named A, B, C and D, are standing in line (A is first, B is second, C is third, D is last). Whenever someone says “boo”, the witches who are first and second in the line switch places. Whenever someone says “trick”, the witches who are second and third in the line switch places. Whenever someone says “treat”, the third and last in the line switch places. In how many ways can the witches rearrange themselves in the opposite order (D, C, B, A) by making the minimal possible number of moves? For example, the following sequence would work:

“Boo! Trick! Treat! Boo! Trick! Boo!”

A team has a better solution to this problem if it is able to present a sequence that the previous team failed to find.

6. (See the attached sheet.) A *snake* is a path of squares that begins at the indicated entrance and does not touch itself, even at corners. Find the longest possible snake on this board.

A team has a better solution to this problem if it is able to present a snake with more squares than used by the previous team.