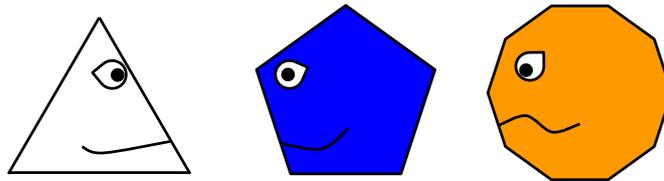


Math Circle - Flatland

source: *The Shape of Space* by Jeffrey R. Weeks

Flatland is a two-dimensional surface. Its only residents live in the tiny city of Flatsburgh. These residents are all two-dimensional beings and can freely move in any direction in Flatland; but before today, they really haven't yet ventured outside the boundaries of Flatsburgh. It's worth restating: the Flatlanders do not move freely *on top of* their surface, but rather they are confined to live completely *within* their two-dimensional land.

Today the Flatlanders are adventurous. They decide to send out 4 expeditions meant to map out as much as they can of their two-dimensional plane. There will be one expedition to the North, one to the East, one South, and one West. The four brave explorers N, E, S, and W are to set out walking precisely in one of the directions each, mapping out as much as they can before turning back to return to Flatsburgh in order to report their findings.



For each of the following four reports, come up with a feasible explanation for the Flatlanders' observations.

N's report: After several days of traveling and beginning to get tired, I was surprised to see S coming straight towards me at a quick pace. I knew I made no mistakes or bad turns in my travels, so I immediately began to fear my hallucination of S – he had set out in the *completely opposite direction*. I could tell by his blank stare at me that S felt the same way.

Turning myself sideways to clear my head, I notice E on my right and W on my left, also perplexedly eyeing our sudden reunion. In disbelief, we sat and ate our lunch together, then silently headed back in a group along the path that S took to reach us. I came back to Flatsburgh on the complete opposite side of town than the side from which I left, and I did not turn around a single time. . .

S's report: My supplies lasted me a week, as I had planned. I was able to chart out some beautiful scenery to the South before turning around and returning to Flatsburgh. I met N on her way back into Flatsburgh, and she also told me of the landscape she discovered in the North before turning back. There is still much more to the South that I was not able to cover, and I'm looking forward to my next expedition!

I do hope that my next expedition is to the South or North, though — I think I would refuse a chance to explore the East or West. It's not that those directions don't seem beautiful, they just seem...strange. Indeed, when I first returned to Flatsburgh, I saw E and W returning through the *opposite* gates! They looked beyond confused. Angry, even. And they absolutely refused to talk to one another.

I talked to E by himself, and he told me that he had actually *seen* W about halfway through his journey! At the time, he ignored W because he imagined that she was just a figment of his imagination — a manifestation of his homesickness. But after returning to Flatsburgh through W's gate, he just didn't know...

E's report: Halfway through my short journey Eastward, I ran straight into W. I wasn't paying much attention to the space in front of me, as I was not expecting to see anyone else there! W was as clearly surprised as I was to find me there, but we both agreed that there must be some good explanation. We continued individually on our own paths and decided to give it some thought. After a little more time, I returned to Flatsburgh, oddly through W's gate.

Neither N nor S returned to Flatsburgh for quite a while after I returned. We were all starting to get worried, when eventually we saw S returning through the North gate and likewise N through the South. N and S had also apparently met halfway along their long journey. I wonder what this all could mean...

W's report: Something weird is going on — I don't like this. All of Flatsburgh is pulling some kind of really strange practical joke on we four explorers.

First of all, I met E on my way journey to the West. At first I wasn't sure it was him. E seemed a little...backwards? Like somehow he was left-is-right and up-is-down...I'm not sure what happened to him, but he seemed to think that *I* was the one who changed. We got into a fight and decided to continue on our separate ways.

Now I *know* that I kept going Westward, but I somehow ended up back here at Flatsburgh. I returned through E's gate, but **everything is different!** Now *all* of Flatsburgh is backwards! At least, everything except myself and the other explorers. E seems back-to-normal to me, and N and S don't look any different — although they tell the same strange story about each other as I reported about E.

I don't like this practical joke one bit...