Math Circle - Winter 2012 - Homework 3

In each problem, make sure you verify the claim to yourself for small n and k.

1. (15 points) Let $n \geq 0$ and $k > 0$ be integers. You are the Dean of Clown College! As the Dean, it is your job to create *balloon boxes* for each of your clowns. Balloons come in $n+1$ different colors, and each balloon box contains k balloons. The k balloons in a balloon box can be any combination of the $n+1$ colors (even all the same color if you want!).

Show that you can make a total of $\binom{n+k}{k}$ different ballon boxes. *Hint.* You can do this with a bijective proof. Recall that $\binom{n+k}{n}$ is the number of ways of choosing n balls from $n+k$ total distinguishable balls. Show that this process is somehow the same as picking different balloon boxes.

2. (10 points) Give a bijective proof (i.e. **not** algebraic manipulation) for:

$$\binom{n}{1} + 2\binom{n}{2} = n^2.$$

Hint. Recall the worksheet from class.

3 (15 points) Prove that the following formula is valid:

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}.$$

Hint. Use a bijective proof – what does the expression on the right count? Also, do not forget that $\binom{n}{k} = \binom{n}{n-k}$.