Think on These Things Week 8

1. Prove that there exist two powers of 2 that differ by a multiple of 2011.

2. We are given 10 integers. Prove that there is always a subset of these whose sum is divisible by 10.

3. A $n \times n$ array of boxes, each box has either a 0 a 1 or a -1. Show that among all the possible sums along rows, columns, and diagonals, two of the sums must be identical.

4. Forty-one rooks are placed on a 10×10 board. Prove that there is a group of five of them that don't attack each other.