

Think on These Things Week 6

1. The San Francisco Math Circle is held in 9 different auditoriums (because they have that many students!). 19 students are in the same grade and attend the same school.
 - (a) Is it true that no matter how these 19 students are distributed amongst the auditoriums there will always be an auditorium with at least three of these students? Why or why not?
 - (b) Must there always be an auditorium with *exactly* 19 students?
2. 12 lines intersect in the point O . Show that it is possible to choose two of them so that the angle between them is less than 17 degrees.
3. A 10×10 board of "Battleship" has one 4-unit battleship (it takes up a 1×4 rectangle). What is the minimal amount of shots you need to guarantee that you will hit the ship at least once?
4. You have a piece of graphing paper that is 100×400 squares. Each square is colored either blue or green. Show that you can always find two horizontal rows and two vertical columns such that their intersections (four squares) are all the same color.
5. A circle is divided into six sections (sound familiar?), the sections have the numbers 1, 0, 1, 0, 0, 0, respectively, in them. You are allowed to add 1 to any neighboring sections. Is it possible to get all of the numbers the same?