1. Let \( \{f_n\} \) be a sequence of analytic functions on a region \( \Omega \) with \( |f_n| \leq 1 \) on \( \Omega \). Let \( K \) be compact and contained in \( \Omega \). Suppose \( \{f_n\} \) converges at infinitely many points in \( K \). Then is it true or false that \( \{f_n\} \) necessarily converges at every point of \( \Omega \)?

2. Let \( F_M \) be the set of functions analytic on the (open) unit disk \( \mathbb{D} \) and continuous on the closed unit disk which satisfy
\[
\int_0^{2\pi} |f(e^{i\theta})|d\theta \leq M.
\]
Show \( F_M \) is a normal family on \( \mathbb{D} \) with respect to the Euclidean metric.

3. Let \( B \) be the set of functions \( f \) which are analytic on the unit disk \( \mathbb{D} \) and satisfy both \( f(0) = 0 \) and \( f(\mathbb{D}) \cap [1, 2] = \emptyset \). Prove \( B \) is a normal family (as maps from \( \mathbb{D} \) into the complex plane with the Euclidean metric) which contains all of its limit functions.

4. Given \( c > 0 \), use normal families to prove there exists an \( r > 0 \) (depending upon \( c \)) such that if \( f \) is analytic on \( \mathbb{D} \), with \( |f(z)| \leq 1 \) for \( z \in \mathbb{D} \) and \( f(0) = 0 \) and \( |f'(0)| > c \), then \( f(\mathbb{D}) \) contains a disk centered at 0 of radius \( r > 0 \). Show the conclusion fails if \( c = 0 \).

5. Do problem 4 without using normal families and obtain an explicit lower bound for \( r \) depending on \( c \).

6. a. Prove that a family \( \mathcal{F} \) of analytic functions on a region \( \omega \) is normal if and only if the family \( \mathcal{F}' = \{f' : f \in \mathcal{F}\} \) is normal and for some \( z_0 \in \Omega \), the set \( \{f(z_0) : f \in \mathcal{F}\} \) is bounded.
   b. Find an example of a sequence \( \{f_n\} \) which is normal on \( \mathbb{C} \) using the spherical metric, but \( \{f'_n\} \) is not normal on \( \mathbb{C} \) using the spherical metric.