Review problems Math 307 Summer 2008

Check your answers carefully. When you draw a graph, be sure to label each curve and give two or three points on the curve.

1. Consider the initial value problem

\[(x - 1)y'' - xy' + y = (x - 1)^2 e^x\]

with \(y(2) = 0\) and \(y'(2) = 3\). Answer the following questions:

a) How many solutions are there and where are the solutions valid? (State why and be sure to show that you have checked the hypotheses of any theorem you use)

b) Note that \(y(x) = e^x\) is a solution to the homogeneous equation (check it)

c) Find the solution to the initial value problem.

d) What is the behavior of the solution for large values of \(x\)? (your answer should have only one term)

2. Suppose a 2240 lb. car compresses a spring 2 inches (1/6 ft). The free vibration of this spring is damped by a shock absorber which applies a force of 2100 lbs if the (vertical) speed is 3 ft/sec. Several members of the JV squad of the local football team push up and down on the bumper with a force equal to 210 \(\sin(10t)\).

a) Describe the motion of the bumper by setting up the differential equation (an initial value problem).

b) Solve the initial value problem

c) Is the bumper underdamped, overdamped or critically damped(why)?

d) What is the maximum displacement (eventually)?

e) What is the quasi-frequency if there is no forcing term?

f) If there were no shock absorber, what force frequency would cause resonance?

g) The captain of the varsity football team walks up to the scene and tells them they can make the car bounce higher by pushing up and down at a different rate. As a good mathematician, he examined the problem abstractly and realized that the frequency of the forcing term that gives the highest amplitude is approximately half way between the answers to part e and part f. What is the maximum displacement (eventually) they will achieve with this frequency? Does the car come off the ground (the spring will no longer be compressed at the maximum vertical displacement.)? (Nb: if the oscillations are big enough, the car will come off the ground and they can then move the car when it is weightless) You may solve the problem by carrying three significant digits instead of using rational or algebraic numbers.

3. A resistor, capacitor and inductor are connected in series with an alternating voltage source (generator). The resistor has resistance 6 ohms, the capacitor has capacitance 0.02 farads, the inductor has inductance 0.5 henries, and the applied voltage is given by \(E(t) = 24\sin(10t)\). Assuming no initial charge on the capacitor, but an initial current of 1 ampere at \(t=0\), when the voltage is first applied,

a) find the subsequent charge.

b) What is the transient term? Graph the transient term.

c) What is the steady state term? (Do not graph)

d) Give initial conditions which would cause either the beat phenomena or resonance if the resistor is removed (one of the two is possible-decide which and give the appropriate initial conditions).

e) Graph your solution to part d).