Name: 

Student number: 

1. (a) Sketch the direction field for 
\[ \frac{dy}{dt} = -4y^2(y - 3)(y - 6). \]
Clearly label the equilibrium solutions, and state whether they are stable, semi-stable or unstable.

(b) Suppose \( y(1) = 2 \). What can you say about the behavior of \( y(t) \) as \( t \) tends to \( +\infty \) by looking at the direction field? Roughly sketch the integral curve which passes through \((1,2)\).

(c) State why your solution never equals 0. If you use a theorem, then clearly state all hypotheses of the theorem and show that you have checked that the hypotheses are satisfied.

2. Solve the initial value problem:
\[ \frac{dy}{dx} = \frac{x(1 + y^2)}{(4 + x^2)} \quad y(2) = 1. \]

3. Do one of the following problems: 3a or 3b. Show all your work. You must write enough to convince me that you didn’t just use a program on your calculator.
\[ y' = 2x - y^2 \quad y(1) = -1. \]

a. Use two steps of Picard’s method with \( y_0(x) = -1 \) to find an approximate solution. What is the value of the approximate solution \( y_2 \) at \( x = 1.2 \)? Clearly label your answer.

OR

b. Use Euler’s method with step size 0.2 beginning at \( x = 1 \) to approximate \( y(1.6) \). Clearly label your answer.

4. Do one of the following problems: 4a or 4b.

a. A cubical block of dry ice rests with one face on a table. Assume that the volume of ice evaporates at a rate proportional to its exposed surface area. If the length of one edge is originally 1 meter and three hours later has been reduced to 0.75 meter, find an expression for the volume at any time. You may assume that as it melts, it still looks like a cubical block (at least until it disappears!). Find the time when the ice disappears. Justify your answer by finding and solving a differential equation.

OR

b. A 4 ohm resistor and an inductor of 1 henry are connected in series with a voltage of \( 100e^{-4t} \cos 50t \). Find the current as a function of time, if there is no current at \( t = 0 \).