1. Let \( f(t) = e^{-t}(\cos(3t) - \sin(3t)) \).
   a. Graph \( f \) for \( t > 0 \).
   b. Find the Laplace transform of \( f \).
   c. Graph the Laplace transform of \( f \) for \( s > 0 \).

2. Find the inverse Laplace transform of the function
   \[
   F(s) = \frac{s}{(s - 3)(s^2 + 2s + 5)}. 
   \]

3. Let \( f \) be the function given by the following:
   \[
   f(t) = \begin{cases} 
   2 \sin(t) & \text{when } 0 < t < \pi/6 \\
   1 & \text{when } \pi/6 < t < \infty 
   \end{cases} 
   \]
   a. Graph \( f \).
   b. Write \( f \) in terms of Heaviside functions.
   c. Solve the following initial value problem using Laplace transforms.
   \[
   y'' + 4y = f \quad \text{with} \quad y(0) = 0, \quad y'(0) = 0. 
   \]
4. The growth rate (per year) of a population of fish in tons is described by the logistic equation

\[ y' = (7 - y)y. \]

The state decides to allow a catch rate of 10 tons per year (i.e. ten tons per year are removed from the population).

a. What ODE does the population satisfy? (DO NOT SOLVE THIS EQUATION EXPLICITLY).

b. What are the critical values?

c. Draw the direction field for your answer to part a.

d. How big should the population be to prevent extinction? (y=0 at extinction).

e. What is the eventual size of the population of fish if y(0) = 3 tons? What is the eventual size of the population of fish if y(0) = 1 ton?

f. Use Euler’s method with step size 0.1 to approximate the solution of the initial value problem if y(0) = 3 tons. Do 5 steps of Euler’s method and plot your answers on the direction field plot. (connect the points with straight line segments).

5. Consider the following electric circuit. There is no charge on the capacitor and no current. At time \( t = 0 \) the switch is then closed.

a. Give an ODE with initial conditions which describes the charge as a function of time.

b. Find the general solution to the Homogeneous equation.

c. Find one solution to the Non Homogeneous equation.

d. Find the general solution to the Non Homogeneous equation.

e. Find the solution to the initial value problem in part a.

f. Graph the solution.

g. Does the solution exhibit a resonance phenomenon, or a beat phenomenon, or neither?