Midterm 1 Math 134 Autumn 2015

Name:

Student Number:

In each problem, justify your answer. You may refer to any theorem we have covered in class or in the book, but clearly indicate which theorems you are using.

1. Suppose n is an integer with $n \geq 2$ and suppose that a and b are real numbers with $a > 0$. Prove that

$$x^{2n} + ax^2 + b = 0$$

has at most two real roots. Hint: consider f''.

2. Suppose a and b are positive real numbers. Find the area of the largest rectangle with sides parallel to the axes contained in the region $\mathcal{R}$ given by

$$\frac{x^4}{a} + \frac{y^4}{b} \leq 1.$$

Your solution should address the questions of why there is a rectangle of maximal area and why the area you found is indeed the largest possible.

Hint: Notice that if (c, d) is a point in the region $\mathcal{R}$ then the rectangle with vertices (c, d), $(-c, d)$, $(-c, -d)$ $(c, -d)$ is contained in $\mathcal{R}$. So we may restrict our attention to rectangles of this form with $c \geq 0,\ d \geq 0$ and (c, d) satisfying the equation

$$\frac{c^4}{a} + \frac{d^4}{b} = 1.$$

3. Prove that a bounded increasing sequence has a limit. In other words: suppose $\{x_n\}$ is a set of real numbers satisfying $x_n < x_{n+1}$ for all positive integers n and suppose there is a real number M so that $x_n \leq M$ for all positive integers n. Prove that there is a real number L so that for each $\varepsilon > 0$ there is an N (depending possibly on ε) so that if $n \geq N$ then $|x_n - L| < \varepsilon$. Hint: L generally won't be equal to M. Draw a picture and make a guess for a good candidate for L.