- Routine problems (don't hand in):

  §2.2. # 7, 9, 10, 13, 18, 41, 42.
  §2.3. # 3, 21, 22, 31, 37, 55.
  §2.4. # 7, 10, 36, 50.
  §2.5. # 16, 27, 43.
  §2.6. # 2, 3, 7, 32.

- To hand in:

(1) (Without using l'Hôpital's Rule) evaluate the limit

$$\lim_{x\to 4}\left\{\frac{\sqrt{x}-2}{(x-4)^2}-\frac{1}{x^2-4x}\right\}$$

(2) Suppose that the function $f:\mathbb{R}\to\mathbb{R}$ has the property that

$$|f(x)-f(y)|\le\frac{1}{2}|x-y|$$

for all $x,y\in(0,1)$.

(a) Prove that f is continuous on $(0,1)$.

(b) Show that if $\lim_{x\to 0^+} f(x)=0$ then the inequality

$$-\frac{1}{2}\le f(x)\le\frac{1}{2}$$

is satisfied for all $x\in(0,1)$.

(3) Suppose that f is continuous on $[0,1]$ and takes values in $[0,1]$, i.e. that for all $x\in[0,1]$, $0\le f(x)\le 1$. Prove that there is a $c\in[0,1]$ such that $f(c)=c$. Such a point is called a *fixed point* of f. **Hints:** *Draw a picture. Consider* $f(x)-x$.

(4) Let n be a positive integer.

(a) Prove that if $0\le a<b$, then $a^n<b^n$.
Hint: *Use mathematical induction.*

(b) Prove that for every nonnegative real number x, there is a unique nonnegative n^{th} root, $x^{1/n}$.
Hint: *The existence follows from the intermediate value theorem. Use part (a) to get uniqueness.*

(5) Suppose $E\subset\mathbb{R}$ has an upper bound. Let

$$\beta=\cup_{p\in\alpha\in E}\{p\}$$

be the set of all rational numbers p that belong to at least one cut $\alpha\in E$. Prove

(a) β is a cut.

(b) β is an upper bound for E.

(c) If x is an upper bound for E then $x\ge\beta$.

The cut β is called the least upper bound for E.