Final Review Math 134 Autumn 2015

1. Divide the interval $[0,2]$ into n subintervals of equal length. Determine a value of n which guarantees an error of less than 10^{-5} if the integral

$$\int_0^2 \sqrt{1+3x}dx$$

is estimated by the

(a) Riemann sums

(b) trapezoidal rule

(c) Simpson's Rule.

2. (a) Using your value of n from problem 1, write down the Simpson's Rule estimate of the integral in problem 1.

(b) Compute the precise value of the integral. Use your calculator to see how close the estimate really is to the true value.

3. Find

$$\int \frac{1}{x^3-1}dx$$

4. Find

$$\int \frac{dx}{5+4\sin x}$$

by making the substitution $u = \tan\frac{x}{2}$, $-\pi < x < \pi$. This idea can be used to convert many integrals involving just trig functions and constants into integrals of rational functions.

5. Prove:

$$\int \arctan x dx = x\arctan x - \frac{1}{2}\ln(1+x^2) + C$$

in two different ways.

6. Find

$$\int \frac{x^2dx}{\sqrt{2x-x^2}}.$$

7. (a) For $\varepsilon > 0$, find the area $A(\varepsilon)$ between the curves given in polar coordinates by $r = \theta$ and $r = \theta + \varepsilon$, $0 \leq \theta \leq 2\pi$.

(b) Compute $\lim_{\varepsilon\to 0} A(\varepsilon)/\varepsilon$.

8. Find the equation of the tangent to the curve given in polar coordinates by $r = 1-2\cos\theta$, when $\theta = \theta_0$. Give your solution in two ways: In Euclidean coordinates and in polar coordinates.