Final Exam Math 134 Autumn 2015

Name:

Student Number:

1. (a) Integrate by parts once:

$$\int x^m e^x dx,$$

$m = 0, 1, 2, \ldots$. There are two ways to integrate by parts. Choose the way which will result in a similar integral but with a lower power of x.

(b) Use your answer to (a) to find (without any further integration) $\int e^x dx$, then $\int xe^x dx$ then $\int x^2 e^x$, then $\int x^3 e^x dx$.

(c) Check your answer to the last integral in (b) using the Fundamental Theorem of Calculus

2. Find the area under the curve given by

$$x(t) = t - \sin t \qquad y(t) = 1 - \cos t,$$

$0 \leq t \leq 2\pi$. To help draw a picture, you might observe that x is increasing and $y \geq 0$.

3. Compute

$$\int_0^1 \frac{12\ dx}{8 - x^3}.$$

4. Consider:

$$\int_{-\pi/3}^{\pi/3} \frac{5 \sin x\ dx}{5 + 4 \sin x + 3 \cos x}.$$

Make the substitution: $u = \tan \frac{x}{2}$ (i.e. $x = 2 \arctan(u)$). This will change the integral into an integral of a rational function. Give the form of the partial fraction expansion and describe in words how you would go about figuring out the unknown coefficients. DO NOT COMPUTE THE COEFFICIENTS and DO NOT COMPUTE THE INTEGRAL.

5. (a) Divide $[0, 1]$ into n subintervals of equal length. Determine a value of n which guarantees an error of less than 10^{-3} if the integral

$$\int_0^1 e^{-x^2} dx$$

is estimated by Simpson's Rule. Your answer should be an explicit integer value for n. Hint: one way to see how large $|p(x)e^{-x^2}|$ is on $[0,1]$ is to note that $e^{-x^2} \leq 1$ then estimate how big $p(x)$ is using calculus. Try a couple of small values of n to see what will work.

(b) Find the estimate of the integral in (a) using Simpson's rule with the value of n you found in part (a). You do not have to simplify your answer, but your answer should only involve rational numbers and the exponential of rational numbers.

6. A bowl in the shape of a hemisphere with diameter 24 inches is filled with water. Below this bowl is an empty bowl in the shape of a paraboloid of revolution which is 16 inches deep and has a top circular opening with diameter 24 inches. (Hint: revolve the curve $y = ax^2$ about the y axis, for an appropriate constant a.) Water leaks out of the bottom of the top bowl into the lower bowl. When the depth of the upper bowl is 4 inches, the depth is decreasing at a rate of one inch per minute. How fast is the depth of the water in the lower bowl rising at this time?