12/7/16 SPRING 2015

6 ⟶

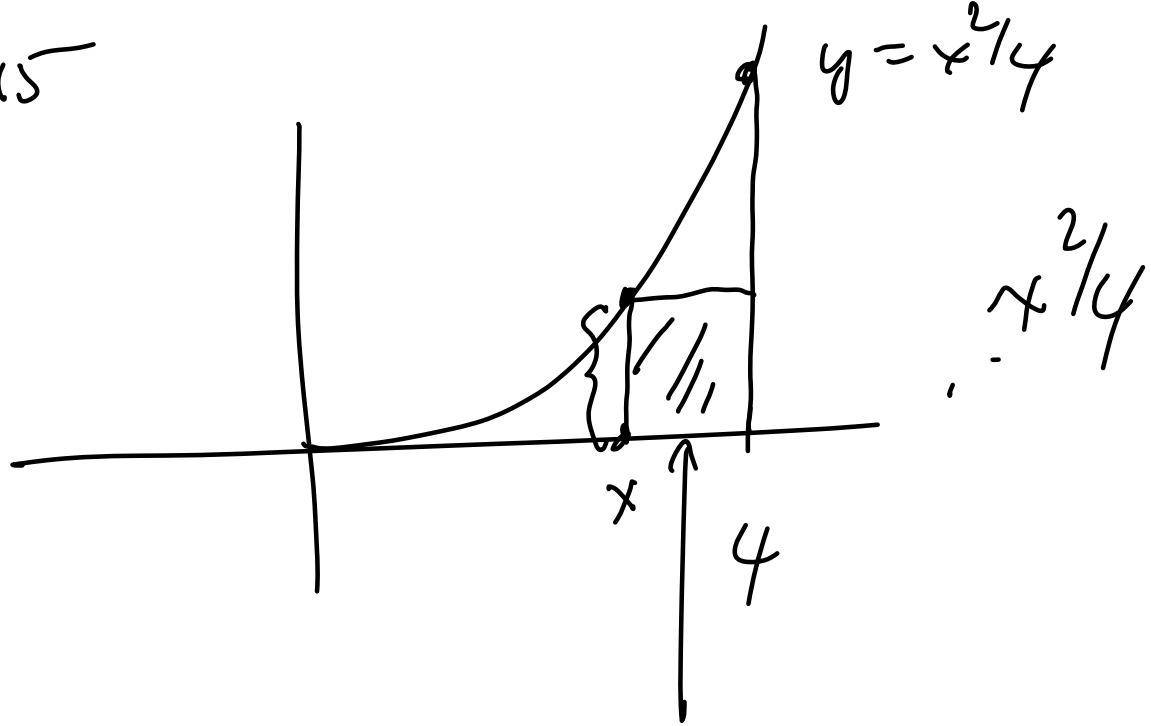
1a

3

4a(iv)

5

2b



$$A(x) = (4-x)\left(\frac{x^2}{4}\right)$$

$$= x^2 - \frac{x^3}{4}$$

$$A'(x) = 2x - \frac{3x^2}{4} = 0$$

$x = 0$

$2 - \frac{3}{4}x = 0$

$x = 8/3$

$A(0) = 0$

$A(4) = 0$

$$A(8/3) = (4 - 8/3)\frac{(8/3)^2}{4}$$

By ext val theorem there is a max & it occurs at $x = 0$ or $x = 8/3$.

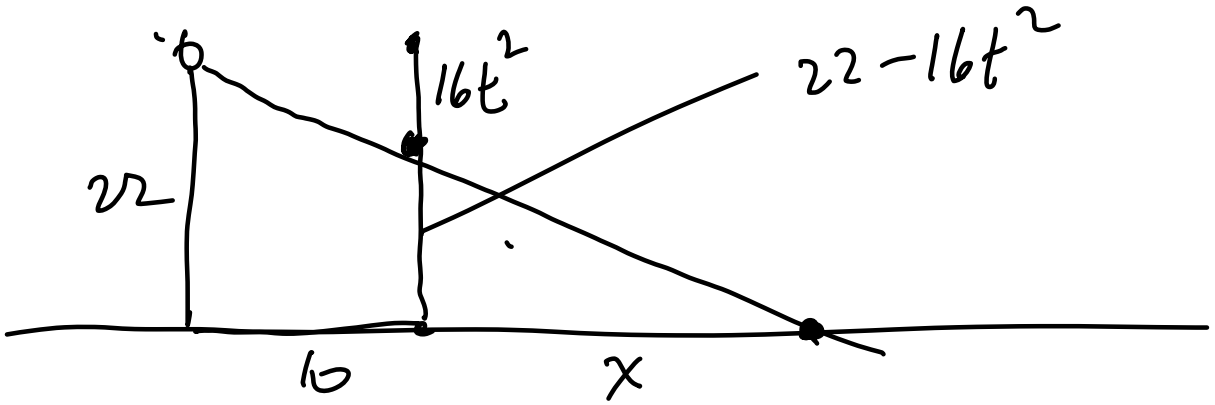
$$\lim_{x \to \infty} x - \sqrt{x^2+4x}$$

$$\left(x - \sqrt{x^2+4x}\right) \frac{\left(x+\sqrt{x^2+4x}\right)}{x+\sqrt{x^2+4x}} = \frac{x^2 - (x^2+4x)}{x+\sqrt{x^2+4x}}$$

$$= \frac{-4x}{x+\sqrt{x^2+4x}} \Rightarrow \frac{-4}{1+\frac{\sqrt{x^2+4x}}{x}}$$

$$= \frac{-4}{\left(1+\frac{\sqrt{x^2(1+4/x)}}{x}\right)}$$

$$= \frac{-4}{1+\sqrt{1+4/x}} \longrightarrow \frac{-4}{2} = -2$$



want $x'\left(\frac{1}{2}\right)$

$$\frac{x}{22-16t^2} = \frac{x+10}{22}$$

$$22x = 22x + 220 - 16t^2x - 160t^2$$

$$16t^2x = 220 - 160t^2$$

$$x = \frac{220-160t^2}{16t^2} = \frac{220}{16t^2} - 10$$

$$x' = -\frac{440}{16t^3}$$

$$x'\left(\frac{1}{2}\right) = \frac{-440}{16\left(\frac{1}{8}\right)} = -220 \text{ ft/sec}$$

$$\frac{\partial}{\partial x} f(x^2) = f'(x^2) \cdot 2x$$

$$\text{at } x=1 \qquad = f'(1) \cdot 2 = -2$$

$x^3 - 4xy + y^2 = 0$

$3x^2 - 4y - 4xy' + 2yy' = 0$

$y'(-4x + 2y) = 4y - 3x^2$

$$y' = \frac{4y - 3x^2}{-4x + 2y}$$

$y'(3) = \frac{4 \cdot 3 - 3 \cdot 9}{-12 + 6} = \frac{-15}{-6} = \frac{5}{2}$

$y = 2x$

If vertical tan $-4x + 2y = 0$

$x^3 - 4x(2x) + (2x)^2 = 0$

$x^3 - 8x^2 + 4x^2 = 0$

$x^3 - 4x^2 = 0$

$x = 0$, $y = 0$

$4y - 3x^2 = 32 - 48 \neq 0$

$x = 4$

$y = 8$

$$x^2 - 4xy + y^2 = 0$$

$$y = \frac{4x \pm \sqrt{16x^2 - 4x^2}}{2}$$

$$= \frac{4x \pm \sqrt{4x^2(4-x)}}{2}$$

$$= \frac{4x \pm 2x\sqrt{4-x}}{2}$$

$$= 2x \pm x\sqrt{4-x}$$

$y' \neq 0$ $\quad y' = 2 \pm \left[\sqrt{4-x} + \frac{x(-1)}{2\sqrt{4-x}}\right]$

bad only if $x = 4$

at (3,3)

$y' = 5/2$

Tang $y = L(x) = \frac{5}{2}(x-3)+3$

$\frac{5}{2}(x-3)+3 = 2.95$

$$x = \frac{t}{t^2+1} \qquad y = \frac{4-t^2}{t^2+1}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\dfrac{(t^2+1)(-2t) - (4-t^2)(2t)}{(t^2+1)^2}}{\dfrac{(t^2+1) - t(2t)}{(t^2+1)^2}}$$

$$= \frac{-2t^3 - 2t - 8t + 2t^3}{t^2+1-2t^2}$$

$$= \frac{-10t}{1-t^2}$$

eq normal

$$y = \frac{1-t^2}{10t}\left(x - \frac{t}{t^2+1}\right) + \frac{4-t^2}{t^2+1}$$

$$0 = \frac{-t(1-t^2)}{10t(t^2+1)} + \frac{4-t^2}{t^2+1}$$

$y^2 - xy + x^2 + y = 22$

$2yy' - y - xy' + 2x + y' = 0$

If $y' = 0$ then $-y + 2x = 0$

$y = 2x$

$4x^2 - 2x^2 + x^2 + 2x = 22$

$3x^2 + 2x - 22 = 0$

$x = \frac{-2 \pm \sqrt{4 + 264}}{6}$

$y = \frac{-2 \pm \sqrt{268}}{3}$