11/30/16

SPRING 2016 FINAL

$f(x)^{g(x)}$ $\qquad x^{1/x}$ $\qquad (1+x)^{\frac{1}{x}}$

$g(x)\underbrace{\ln f(x)}$

$g(x) \to 0$, $\ln f(x) \to \infty$

0

$$\frac{\ln f(x)}{\left(1/g(x)\right)} \qquad \ln f(x) \to \infty, \quad 1/g(x) \to \infty$$

$$\frac{g(x)}{\frac{1}{\ln f(x)}} \qquad g(x) \to 0, \quad \frac{1}{\ln f(x)} \to 0$$

$$\left(1-\frac{1}{x^2}\right)^x$$

$$x \ln\left(1-1/x^2\right)$$

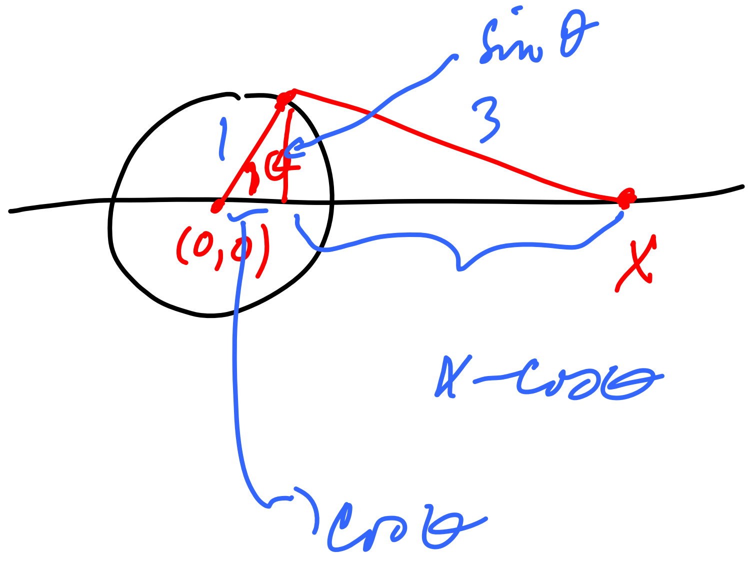
$$\frac{\ln\left(1-1/x^2\right)}{1/x}$$

6

36

4

19



$$3^2 = \sin^2\theta + (x-\cos\theta)^2$$

want $\frac{d\theta}{dt}$.

when $\theta = \pi/3$ $\quad \frac{dx}{dt} = \frac{1}{2}$

$$9 = \underbrace{\sin^2\theta + x^2 - 2x\cos\theta + \cos^2\theta}_{1}$$

$$8 = x^2 - 2x\cos\theta$$

$$0 = 2x\frac{dx}{dt} - 2\left[\frac{dx}{dt}\cos\theta - x\sin\theta\frac{d\theta}{dt}\right]$$

$\theta = \pi/3$

$$8 = x^2 - 2(x)\tfrac{1}{2}$$

$$0 = x^2 - x - 8$$

$$x = \frac{1 \pm \sqrt{1+32}}{2} = \frac{1+\sqrt{33}}{2}$$

$x = 3t^2+1 \qquad y = 2t^3+1$

Fix t slope

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t^2}{6t} = t$$

Tang line $\quad y = t\left(x-(3t^2+1)\right) + 2t^3+1$

hits $(5,1)$ if:

$$\cancel{1} = t\left(5-(3t^2+1)\right) + 2t^3 \cancel{+1}$$

$$0 = 5t - 3t^3 - t + 2t^3$$

$$0 = 4t - t^3$$

$t=0$ or $4-t^2=0 \qquad t = \pm 2$

if $t=0 \quad y=1$ (for all x)

if $t=2 \quad y = 2(x-13)+17$

$t=-2 \quad y = -2(x-13)-15$

$$\sin(x+y) + \cos(x+y) = e^{3x} \qquad y(0)=0$$

$$[\cos(x+y)][1+y'] + -\sin(x+y)[1+y'] = 3e^{3x}$$

want $y'(0)$;
$y(0)=0$

$$\underbrace{\cos(0+0)}_{1}(1+y'(0)) - \sin(0+0)[1+y'(0)] = 3e^{0}$$

$$1+y'(0) = 3$$

$$y'(0) = 2$$

Est $f(-0.1)$

Tangent line $y = 2x$ — tangent line at -0.1)

$$y(-0.1) = -0.2 \approx \underline{f(-0.1)}$$

Find $y''(0)$;

$$-\sin(x+y)[1+y']^2 + [\cos(x+y)]y'' - [\cos(x+y)][1+y']^2 - \sin(x+y)\,y'' = 9e^{3x}$$

PLUG IN $x=0$, $y=0$ $y'(0)=2$ solve.

L'Hospital's Rule

$$\lim_{x\to a} \frac{f(x)}{g(x)}$$

1. $f \to 0$ as $x \to a$
2. $g \to 0$ as $x \to a$
3. $\frac{f'(x)}{g'(x)} \to L$ as $x \to a$
4. $g'(x) \neq 0$ x near a
   ↳ value at a doesn't matter

Then $\frac{f(x)}{g(x)} \to L$

~~$a=0$~~

let $x \to \infty$

$f'(x) = \sin x + 1$ $\qquad f(x) = \cos x + x$

$g'(x) = 2 + \cos x$ $\qquad g = 2x + \sin x$

$$\frac{f}{g} = \frac{\cancel{x}\left[\frac{\cos x}{x} + 1\right)}{\cancel{x}\left[2 + \frac{\sin x}{x}\right)} \longrightarrow 1/2$$

$$f'(x) = \frac{\sin^2 x}{x^{2.5}}$$

$$g'(x) = \frac{\sin x}{x^2}$$

$$f(x) = -\int_x^\infty \frac{\sin^2 t}{t^{2.5}}\,dt \qquad g(x) = -\int_x^\infty \frac{\sin t}{t^2}\,dt$$

$f \to 0$

$g \to 0$

$$\frac{f'}{g'} = \frac{\sin^2 x}{x^{2.5}} \cdot \frac{x^2}{\sin x} = \frac{\sin x}{\sqrt{x}}$$

$\longrightarrow 0$

$f/g \longrightarrow \infty$ as $x \to \infty$

$$\lim_{x \to 0^+} x \ln x$$

$$\frac{\ln x}{\frac{1}{x}}$$

f g

① $f' = 1/x \to \infty$

② $g' = -1/x^2 \to -\infty$

④ $\neq 0$

$$f'/g' = \frac{1/x}{-1/x^2} = -x$$

③ $\to 0$

so $\frac{f}{g} \to 0$