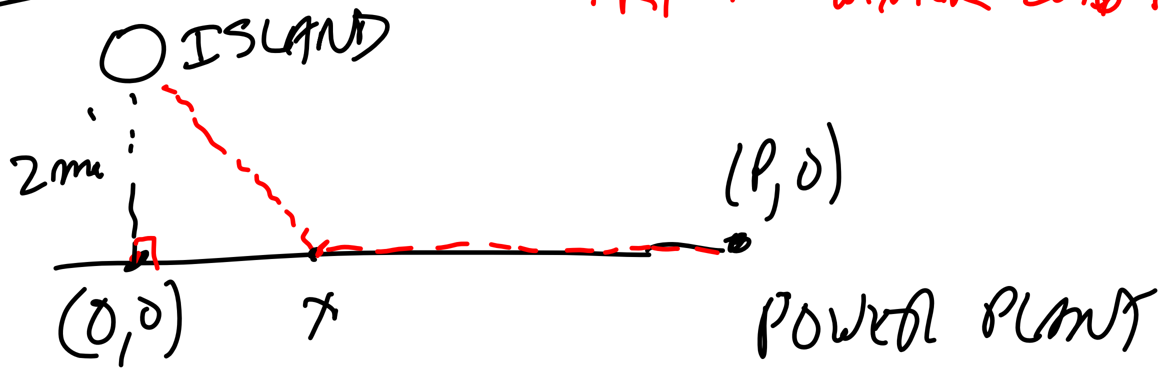
11/28/16

FOR WEDS: SPRING 2016 FINAL

FRI: WINTER 2016 FINAL



$0 \le x \le P$

Cable costs \$2400/mi in water

\$1200/mi on land

CHEAPEST CABLE? JUSTIFY SOLUTION.

$$C = 2400\sqrt{x^2+4} + 1200[P-x]$$

$$C'(x) = \frac{2400}{\cancel{2}}\,\frac{\cancel{2}x}{\sqrt{x^2+4}} - 1200 = 0$$

$$\frac{2x}{\sqrt{x^2+4}} - 1 = 0$$

$$2x - \sqrt{x^2+4} = 0$$

$$2x = \sqrt{x^2+4}$$

$$4x^2 = x^2+4$$

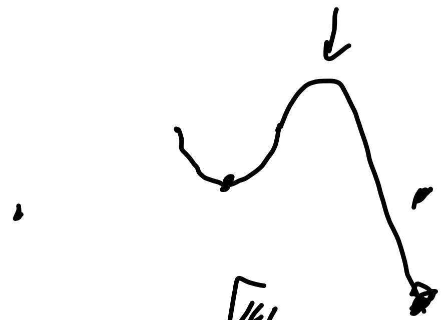
$$3x^2 = 4$$

$$x^2 = 4/3$$

$$x = \sqrt{4/3}$$

MIN COST

$$C(\sqrt{4/3}) = \left(2400\sqrt{4/3+4} + 1200[P-\sqrt{4/3}]\right)$$

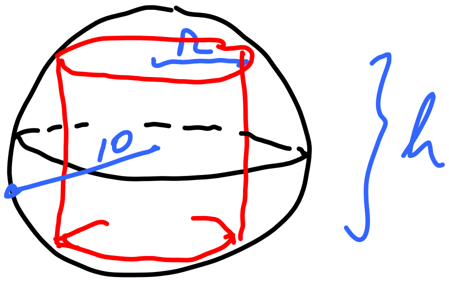


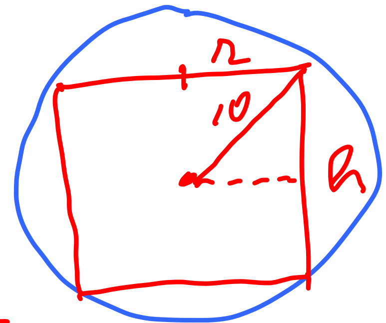
$\sqrt{4/3}$

$\Uparrow$

only cp,

Find HEIGHT h & RADIUS r of CYLINDER WITH MAX VOLUME IN SPHERE OF RADIUS 10.





$0 \le h \le 20$

$$r^2 + \left(\frac{h}{2}\right)^2 = 10^2$$

$$Vol = \pi r^2 h$$

$$V(h) = \pi\left(100 - \frac{h^2}{4}\right)h = 100\pi h - \frac{\pi h^3}{4}$$

$$V'(h) = 100\pi - \frac{3\pi}{4}h^2 = 0$$

$$\boxed{\frac{400}{3} = h^2}$$

$$h = \frac{20}{\sqrt{3}}$$

$$r^2 = 100 - \left(\frac{10}{\sqrt{3}}\right)^2 = 100 - \frac{100}{3} = \frac{200}{3}$$

$$r = \sqrt{\frac{200}{3}}$$

$$V(0) = 0$$

$$V(20) = 100\pi \cdot 20 - \frac{\pi \cdot 8 \cdot 1000}{4} = 0$$

MUST BE MAX. $\rightarrow$ $V\left(\frac{20}{\sqrt{3}}\right) = \pi\left(\frac{200}{3}\right)\frac{20}{\sqrt{3}}$

EXAMPLE: GRAPH:

$$f(x) = 1 + \frac{1}{x} - \frac{1}{x^2} = \frac{1}{x^2}\left[x^2 + x - 1\right]$$

DOMAIN: $x \neq 0$

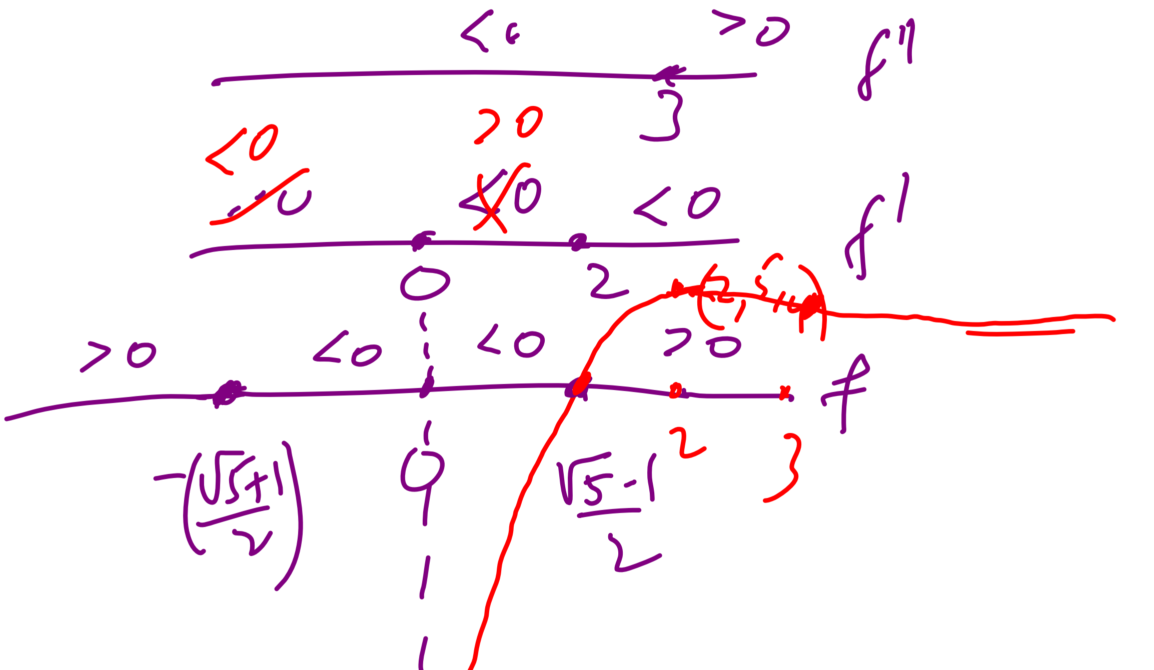
$$f'(x) = -\frac{1}{x^2} + \frac{2}{x^3} = -\frac{1}{x^3}\left[x - 2\right]$$

cp at $x = 2$

$$f''(x) = \frac{2}{x^3} - \frac{6}{x^4} = \frac{2}{x^4}\left[x - 3\right]$$

$f'' > 0$ if $x > 3$

$f'' < 0$ if $x < 3$



$f(3) = 1 + \frac{1}{3} - \frac{1}{9} = \frac{11}{9}$

$f(2) = 1 + \frac{1}{2} - \frac{1}{4} = 5/4$

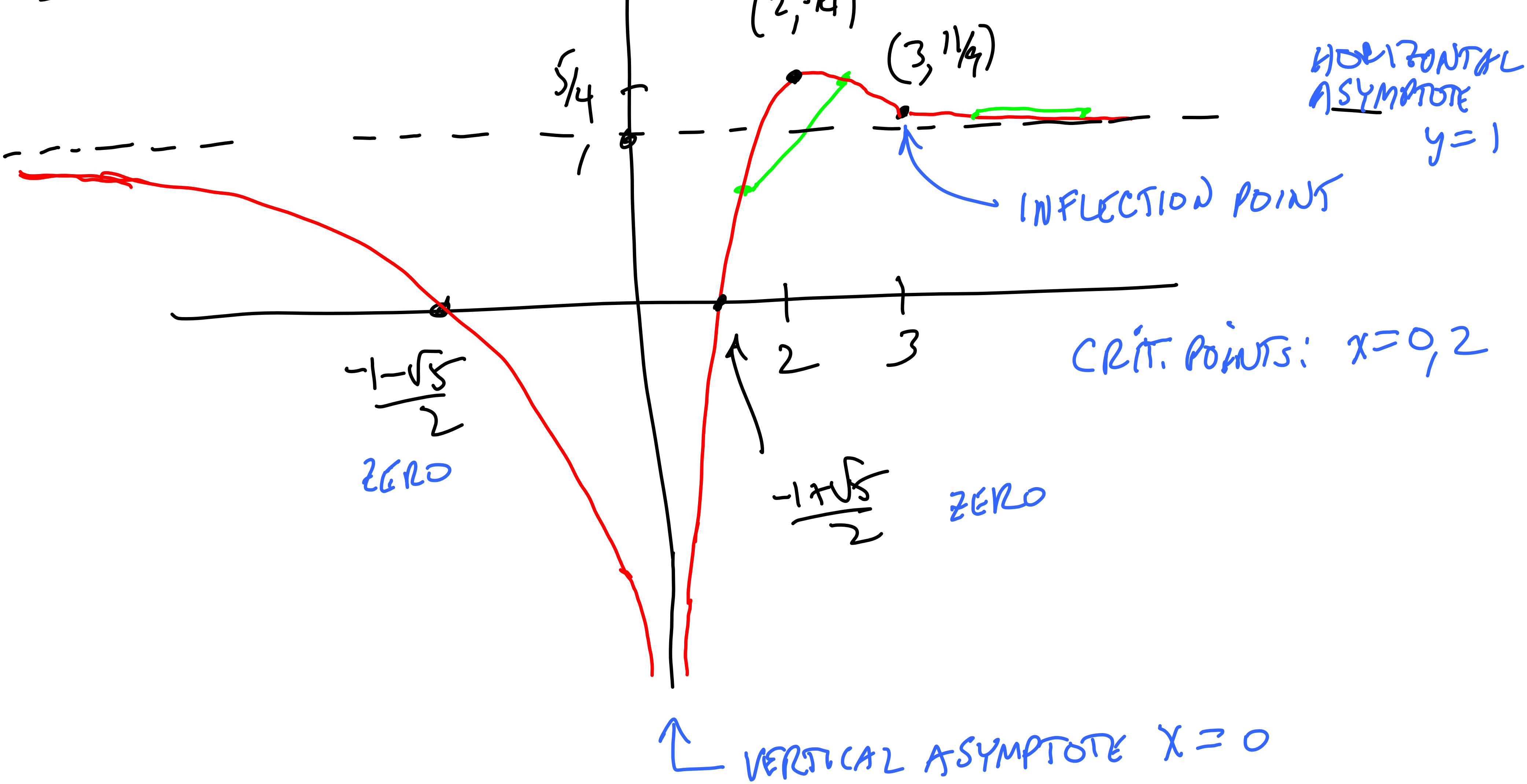
$x^2 + x - 1 = 0$

$$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\lim_{x \to \infty} f(x) = 1$$

near 0 $f(x) \sim -\frac{1}{x^2}$ so $\lim_{x \to 0} f(x) = -\infty$

GRAPH



INFLECTION POINTS ARE <u>ON</u> THE GRAPH – CONCAVITY CHANGES AT 0 BUT NO INFL. POINT THERE