11/23/16

4.5 Done except now can use L'Hospital's Rule to find horizontal asymptotes and determine behavior near places where f is undefined. [skip slant asymptotes]

Graph

$$f(x) = \frac{e^x - 1}{x} \quad \text{near } x = 0$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

$$\left\{ \begin{array}{l} e^x - 1 \to 0 \\ x \to 0 \\ \frac{e^x}{1} \to 1. \\ \frac{d}{dx} x \neq 0 \end{array} \right.$$

$$L'H \Rightarrow \frac{e^x - 1}{x} \to 1.$$

extend the def of f so that $f(0)=1$ i.e. $g(x)=\begin{cases} f(x) & x\neq 0 \\ 1 & x=0 \end{cases}$

continuous

Deriv at $x=0$?

$\forall x\neq 0$

$f=\frac{e^x-1}{x}$

$$g'(x)=f'(x)=\frac{xe^x-(e^x-1)}{x^2}=\boxed{\frac{(x-1)e^x+1}{x^2}}$$

$$\frac{g(h)-g(0)}{h}=\frac{\left[\frac{e^h-1}{h}-1\right]}{h}=\frac{e^h-1-h}{h^2}$$

$\downarrow$ as $h\to 0$

$\frac{1}{2}$

as $h\to 0$:

$e^h-1-h\to 0$

$h^2\to 0$

$\frac{e^h-1}{2h}\longrightarrow\frac{1}{2}$

$h\neq 0$ h near 0 (except at 0)

To find $g''(0)$

want $\lim_{h \to 0} \dfrac{g'(h) - g'(0)}{h}$

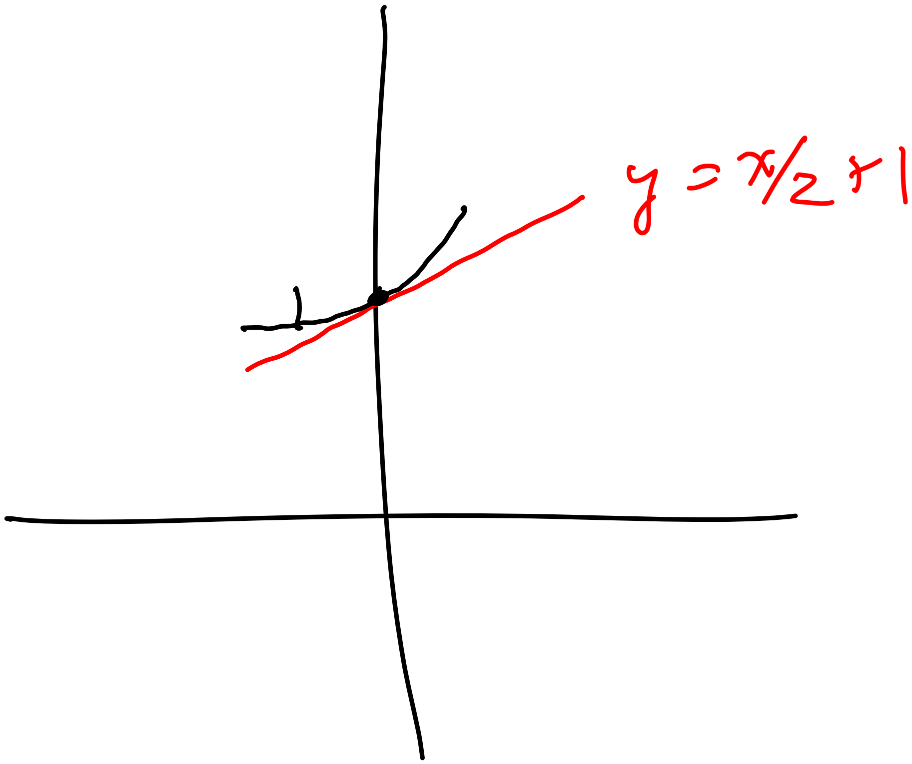
$$\frac{g'(h) - g'(0)}{h} = \frac{\frac{(h-1)e^h + 1}{h^2} - \frac{1}{2}}{h} = \frac{(h-1)e^h + 1 - h^2/2}{h^3}$$

$(h-1)e^h + 1 - h^2/2 \longrightarrow 0$ as $h \to 0$

$h^3 \to 0$ as $h \to 0$ (not 0 except at 0)

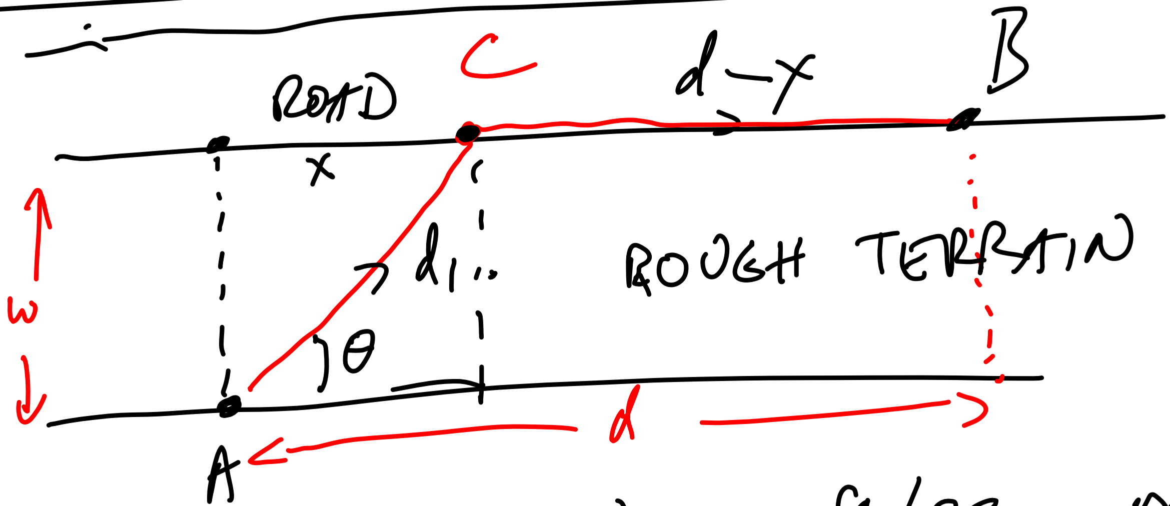
$$\frac{e^h + (h-1)e^h - h}{3h^2} = \frac{he^h - h}{3h^2} = \frac{e^h - 1}{3h} \longrightarrow 1/3$$

So $g''(0) = 1/3$ concave up


$y = x/2 + 1$

4.7

ORIENTEERING



FIND C TO GET TO B AS FAST AS POSSIBLE

RUN 10 ft/sec on ROAD
4 ft/sec on ROUGH TERRAIN

SPEED · TIME = DISTANCE

TIME = DIST / SPEED (CONST SPEED)

$$\frac{d_1}{4\,\text{ft/sec}} + \frac{d-x}{10\,\text{ft/sec}}$$

$\sin\theta = \frac{w}{d_1}$ $d_1 = w\csc\theta$ $\frac{x}{d_1} = \cos\theta$ $x = w\csc\theta\cos\theta$

$$T(\theta) = \frac{w\csc\theta}{4} + \frac{d - w\csc\theta\cos\theta}{10} = \frac{d}{10} + w\csc\theta\left[\frac{1}{4} - \frac{\cos\theta}{10}\right]$$

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$$T(\theta) = \frac{d}{10} + w\csc\theta\left[\frac{1}{4} - \frac{\cos\theta}{10}\right]$$

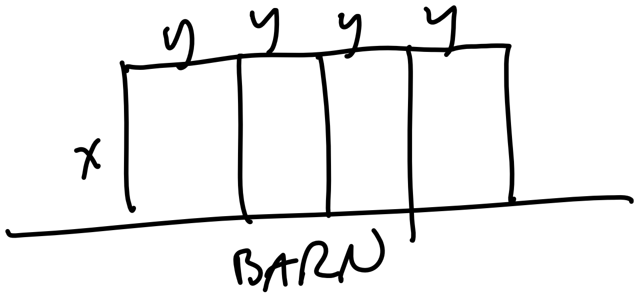
$$T'(\theta) = w\left\{\left[-\csc\theta\cot\theta\right]\left[\frac{1}{4} - \frac{\cos\theta}{10}\right] + \csc\theta\left[\frac{\sin\theta}{10}\right]\right\} = 0$$

$$-\frac{\cos\theta}{\sin^2\theta}\left(\frac{1}{4} - \frac{\cos\theta}{10}\right) + \frac{1}{10} = 0$$

$$-\frac{\cos\theta}{4} + \frac{\cos^2\theta}{10} + \frac{\sin^2\theta}{10} = 0$$

$$-\frac{\cos\theta}{4} + \frac{1}{10} = 0$$

$$2/5 = \frac{4}{10} = \cos\theta \qquad \theta = \cos^{-1}(2/5)$$



4 HOG PENS AREA 100 ft^2 EACH

MIN LENGTH OF FENCE

$$xy = 100 \qquad y = \frac{100}{x}$$

$$L = 5x + 4y$$

$$L = 5x + \frac{400}{x}$$

$$L' = 5 - \frac{400}{x^2} = 0$$

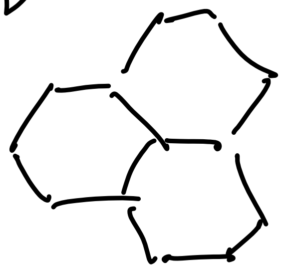
$$5 = \frac{400}{x^2}$$

$$x^2 = 80 \qquad x = 4\sqrt{5}\ldots$$

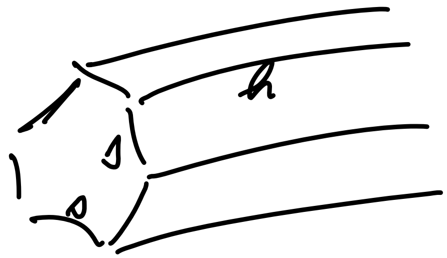
$$L(4\sqrt{5}) = 5 \cdot 4\sqrt{5} + \frac{400}{4\sqrt{5}} = 20\sqrt{5} + \frac{100}{\sqrt{5}}$$

$$= 20\sqrt{5} + 20\sqrt{5} = 40\sqrt{5}$$

BEE HIVE

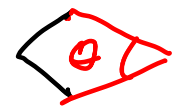


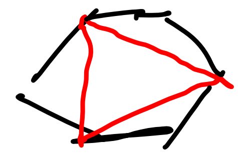
HEXAGONS



ENDS SLICED OFF

3 CUTS





SURFACE AREA

$$S = 6sh - \frac{3}{2}s^2\left[\cot\theta - \sqrt{3}\csc\theta\right]$$

WHAT θ GIVES MINIMAL SURFACE AREA?

$$\frac{dS}{d\theta} = -\frac{3}{2}s^2\left[-\csc^2\theta - \sqrt{3}\left(-\csc\theta\cot\theta\right)\right]$$

$$= 0$$

$$\sqrt{3}\,\frac{\cos\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\cos\theta = 1/\sqrt{3}$$

$\theta \sim 55^\circ$