11/21 A COUPLE OF SMALL COMMENTS: OH: 1:30-2:20 M/W.

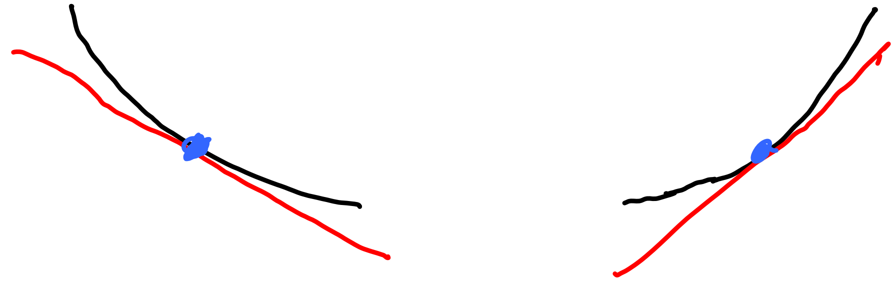
$$\lim_{x\to 0} \frac{\tan 3x}{\sin 2x}$$

(1) $\tan 3x \to 0$
(2) $\sin 2x \to 0$) as $x \to 0$

(3) $\dfrac{\sec^2(3x)\cdot 3}{2\cos(2x)} \longrightarrow \dfrac{3}{2}$

(4) $2\cos(2x) \neq 0$ near 0. (except pos. at 0)

Then $\displaystyle\lim_{x\to 0} \frac{\tan 3x}{\sin 2x} = 3/2$

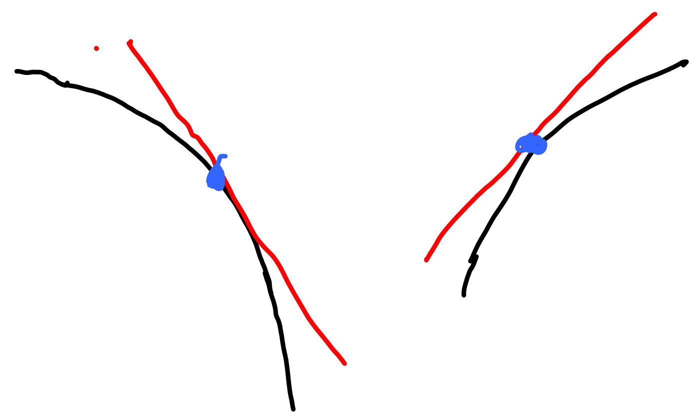


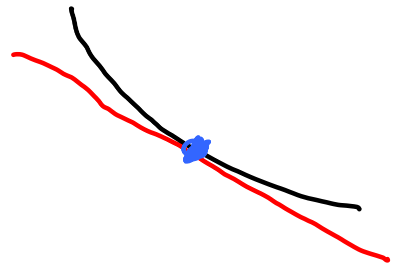
CONCAVE UP ON AN INTERVAL:

- ALL TANGENT LINES BELOW GRAPH
- f' INCREASING
- $f'' \geq 0$ (IF DIFFERENTIABLE)

CONCAVE DOWN ON AN INTERVAL:

- ALL TANGENT LINES ABOVE GRAPH
- f' DECREASING
- $f'' \leq 0$ (IF DIFFERENTIABLE)





CONCAVE UP ON AN INTERVAL:

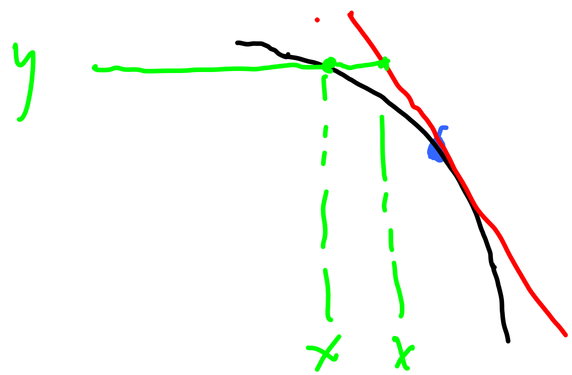
- ALL TANGENT LINES BELOW GRAPH
- f' INCREASING
- $f'' > 0$ (IF DIFFERENTIABLE)

LINEAR APPROX IS SMALLER THAN FUNCTION

CONCAVE DOWN ON AN INTERVAL:

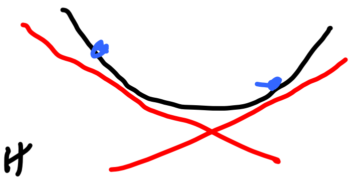
- ALL TANGENT LINES ABOVE GRAPH
- f' DECREASING
- $f'' < 0$ (IF DIFFERENTIABLE)

LINEAR APPROX IS LARGER THAN FUNCTION



WHAT ABOUT INVERSE OF LINEAR FUNCTION?

CONCAVE UP ON AN INTERVAL:

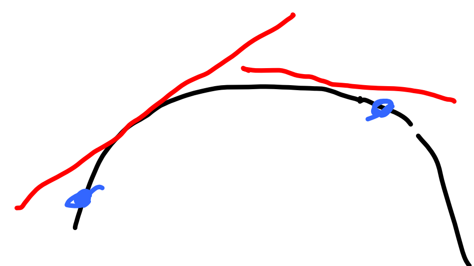


LINEAR APPROX IS SMALLER THAN FUNCTION

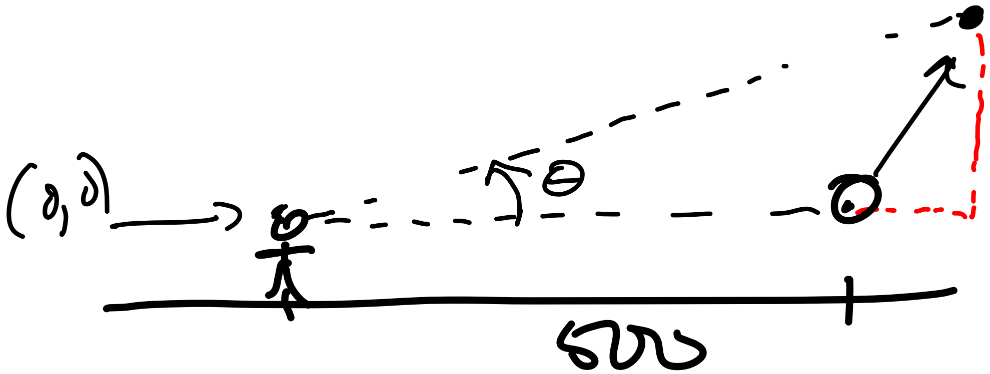
- ALL TANGENT LINES BELOW GRAPH
- f' INCREASING
- $f'' > 0$ (IF DIFFERENTIABLE)
- CHORDS ABOVE GRAPH

CONCAVE DOWN ON AN INTERVAL:

- ALL TANGENT LINES ABOVE GRAPH
- f' DECREASING
- $f'' < 0$ (IF DIFFERENTIABLE)
- CHORDS BELOW GRAPH



LINEAR APPROX IS LARGER THAN FUNCTION



vert 100 ft/min
horiz 75 ft/min

t = 0 release balloon

$x(t) = 500 + 75t$

$y(t) = 0 + 100t$

$$\tan\theta = \frac{100t}{500+75t}$$

$$\sec^2\theta \, \frac{d\theta}{dt} = (\quad)'$$

$$x^{\frac{1}{x}} = e^{(\ln x)\cdot\frac{1}{x}}$$

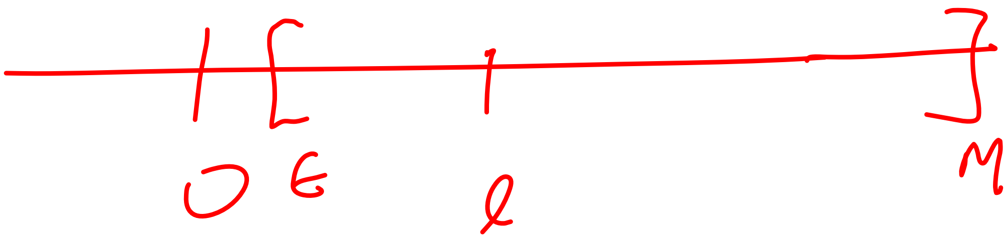
$$\frac{\ln x}{x} \longrightarrow 0 \quad \text{as } x \to \infty$$

$$\frac{\ln x}{x} \longrightarrow -\infty \quad \text{as } x \to 0^+$$

$$x^{\frac{1}{x}} \longrightarrow \begin{cases} 1 & \text{as } x \to \infty \\ 0 & \text{as } x \to 0 \end{cases}$$

loc. max at $x = e$

$e^{1/e}$



$$e^{\frac{1}{e}} > 1$$

<u>One</u> max occurs at e

so $x^{1/x} \le e^{1/e}$ $\quad (x > 0)$

$$\pi^{1/\pi} \le e^{1/e}$$

$$y = x^{1/x}$$

$$\ln y = \frac{1}{x}\ln x$$

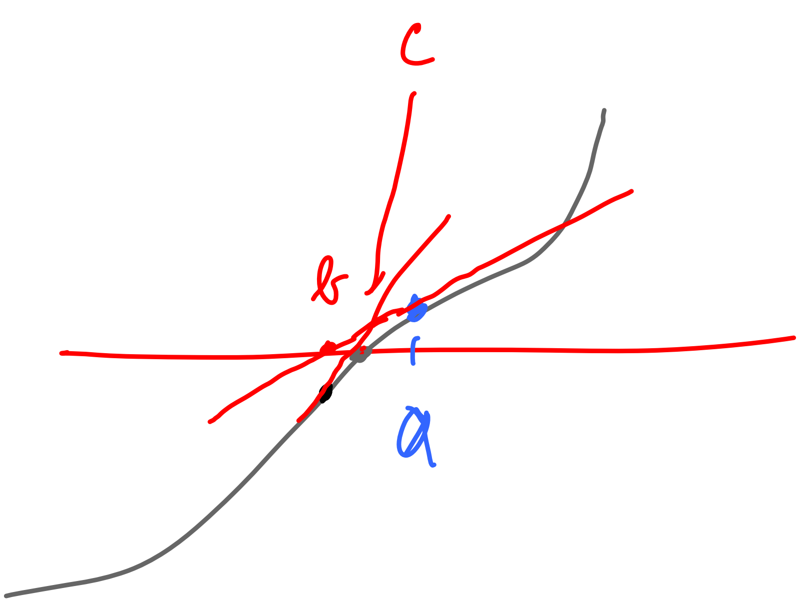
$$\frac{y'}{y} = \frac{x\cdot\frac{1}{x} - \ln x}{x^2}$$

$\ln x = 1$ $(x = e)$

$$\pi^{\frac{1}{\pi}} \le e^{1/e}$$

$$\left[\pi^{\frac{1}{\pi}}\right]^{\pi e} \le \left[e^{1/e}\right]^{\pi e}$$

$$\pi^{e} \le e^{\pi}$$



want to find x so that $f(x)=0$

Guess $x \approx a$

Linearize at a;

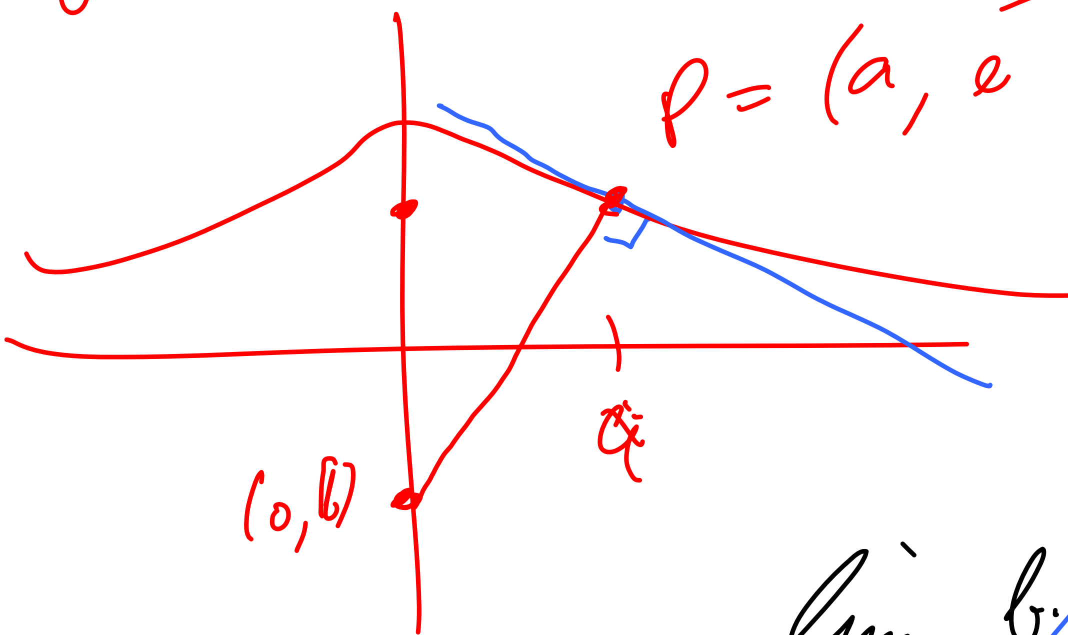
$f(a)+f'(a)(x-a)$

Solve $0=f(a)+f'(a)(x-a)$

for x new value $x=b$

$$y = e^{-x^2/17}$$

$$P = \left(a,\ e^{-a^2/17}\right)$$



$$\lim_{a \to 0} b.$$

$$y' = -\frac{2x}{17} e^{-x^2/17}$$

at $x = a$;

tang line

$$L(x) = -\frac{2a}{17} e^{-a^2/17}(x-a) + e^{-a^2/17}$$

normal line

$$N(x) = \frac{17}{2a} e^{a^2/17}(x-a) + e^{-a^2/17}$$

$$b = \frac{17}{2a} e^{a^2/17}(-a) + e^{-a^2/17}$$

$$= -\frac{17}{2} e^{a^2/17} + e^{-a^2/17}$$

$$\xrightarrow[a \to 0]{} -17/2 + 1$$