11/14/16 $f'(x) > 0$ on an interval $\Rightarrow$ f increasing on the interval

$f'(x) < 0$ on an interval $\Rightarrow$ f decreasing on the interval

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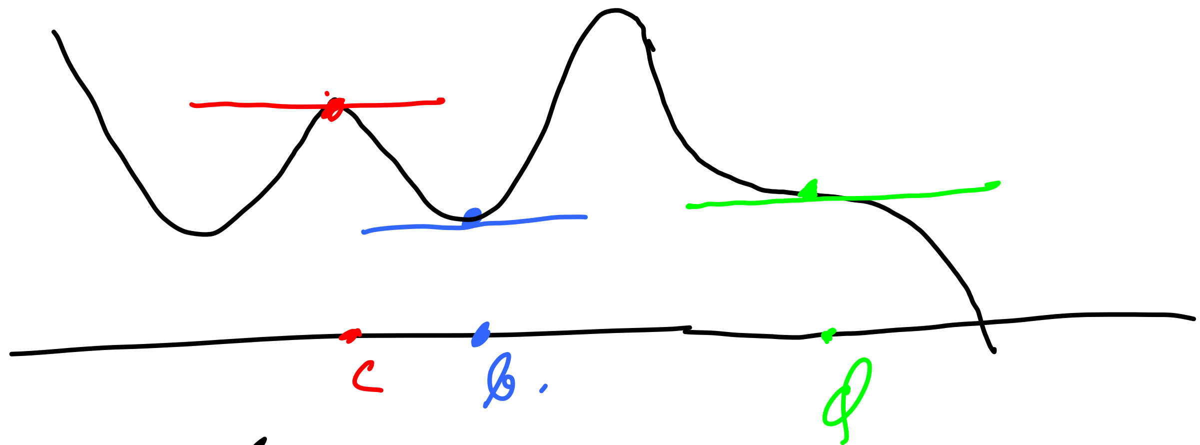


c is local max

- Notice $f'(x)$ changes from pos to neg as x crosses c from left to right
- $f'(x)$ changes from neg. to pos. as x crosses b from left to right
- f' does not change sign at d

First derivative test

SUPPOSE $f'(c)=0$

LOCAL MAX AT C: f' CHANGES FROM POS. TO NEG. AT C

LOCAL MIN AT C: f' CHANGES FROM NEG TO POS AT C

NO LOC MIN OR MAX AT C: f' DOES NOT CHANGE SIGN AT C

USEFUL: If f is CONTINUOUS on $[a,b]$ then f' CAN ONLY CHANGE SIGN AT A CRITICAL NUMBER

EXAMPLE: $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

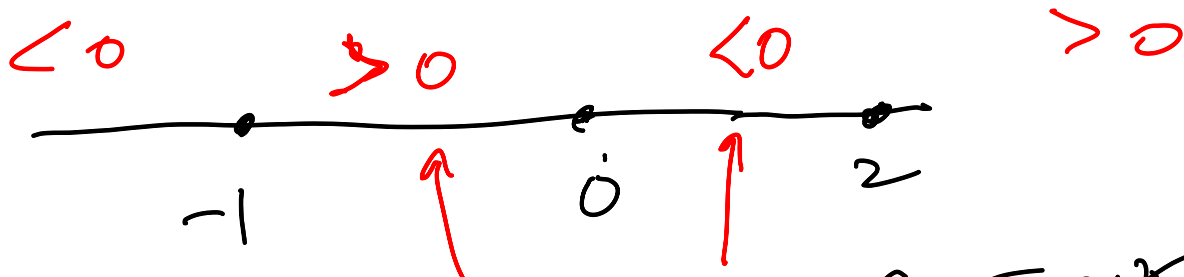
$$f'(x) = 12x^3 - 12x^2 - 24x = 12x\left[x^2 - x - 2\right]$$
$$= 12x\left[x-2\right]\left[x+1\right]$$

CRITICAL NUMBERS: $0, 2, -1$

f':	<0	-1	>0	0	<0	2	>0

1. CHECK AS $x \to \pm\infty$.
2. CHECK FOR SIGN CHANGES AT C.N.

ANOTHER WAY:



CHECK VALUE AT ONE POINT IN THE INTERVAL

$f'(x) = 12x(x-2)(x+1)$

$f'(1) = 12(-1)\cdot 2 < 0$

So $f' < 0$ in $(0,2)$

$f'(-1/2) = 12(-1/2)\left(-\frac{1}{2}-2\right)\left(-\frac{1}{2}+1\right)$

neg neg pos

pos.

So $f' > 0$ in $(-1,0)$

WHY USEFUL?

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x(x-2)(x+1)$

f': <0 | -1 | >0 | 0 | <0 | 2 | >0

f' changes from <0 to >0 at -1 so LOCAL MIN AT -1

>0 to <0 at 0 so LOCAL MAX AT 0

<0 to >0 at 2 so LOCAL MIN AT 2

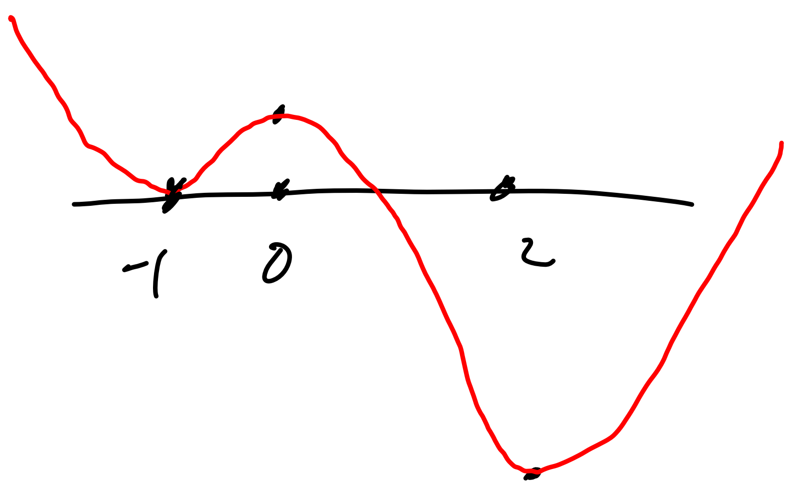
$f(-1) = 3 + 4 - 12 + 5 = 0$

$f(0) = 5$

$f(2) = 3 \cdot 16 - 4 \cdot 8 - 12 \cdot 4 + 5$

$= -32 + 5 = -27$

GRAPH OF f:

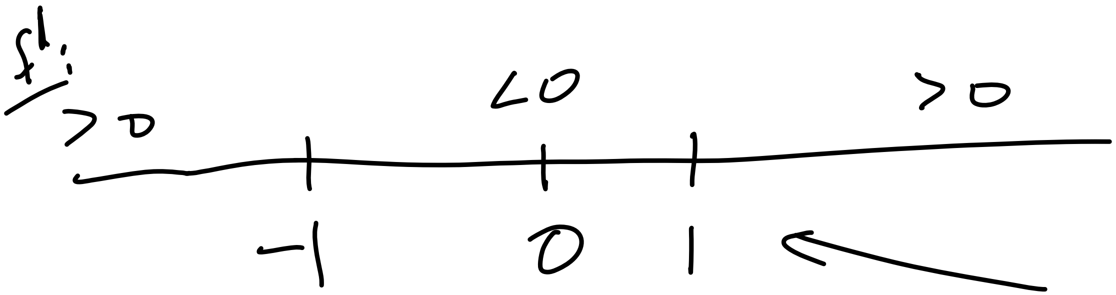


EXAMPLE

$$f(x) = x - 2\arctan x$$

$$f'(x) = 1 - \frac{2}{1+x^2} = 0 \qquad 1+x^2 = 2 \qquad x = \pm 1$$

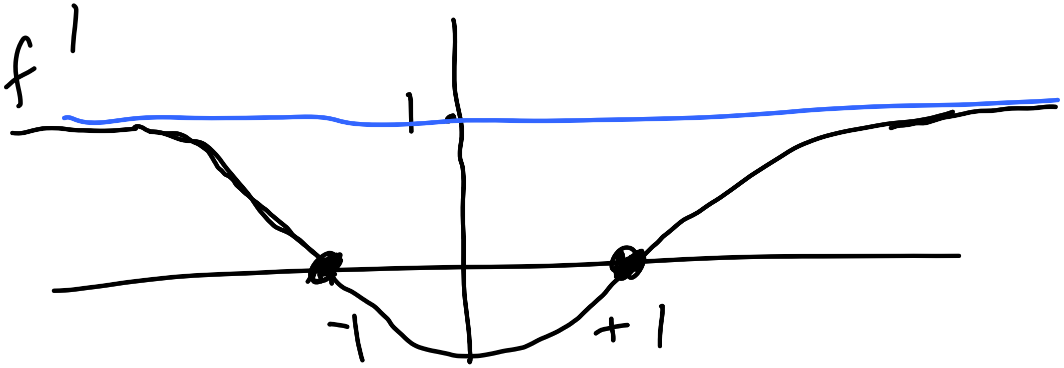
[f' changes sign at ± 1
or use: $f'(0) = -1$]



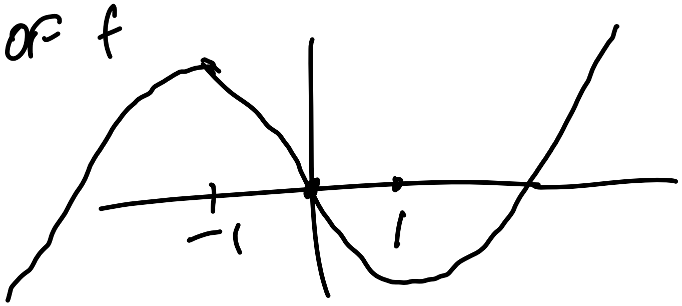
LOCAL MIN AT +1.

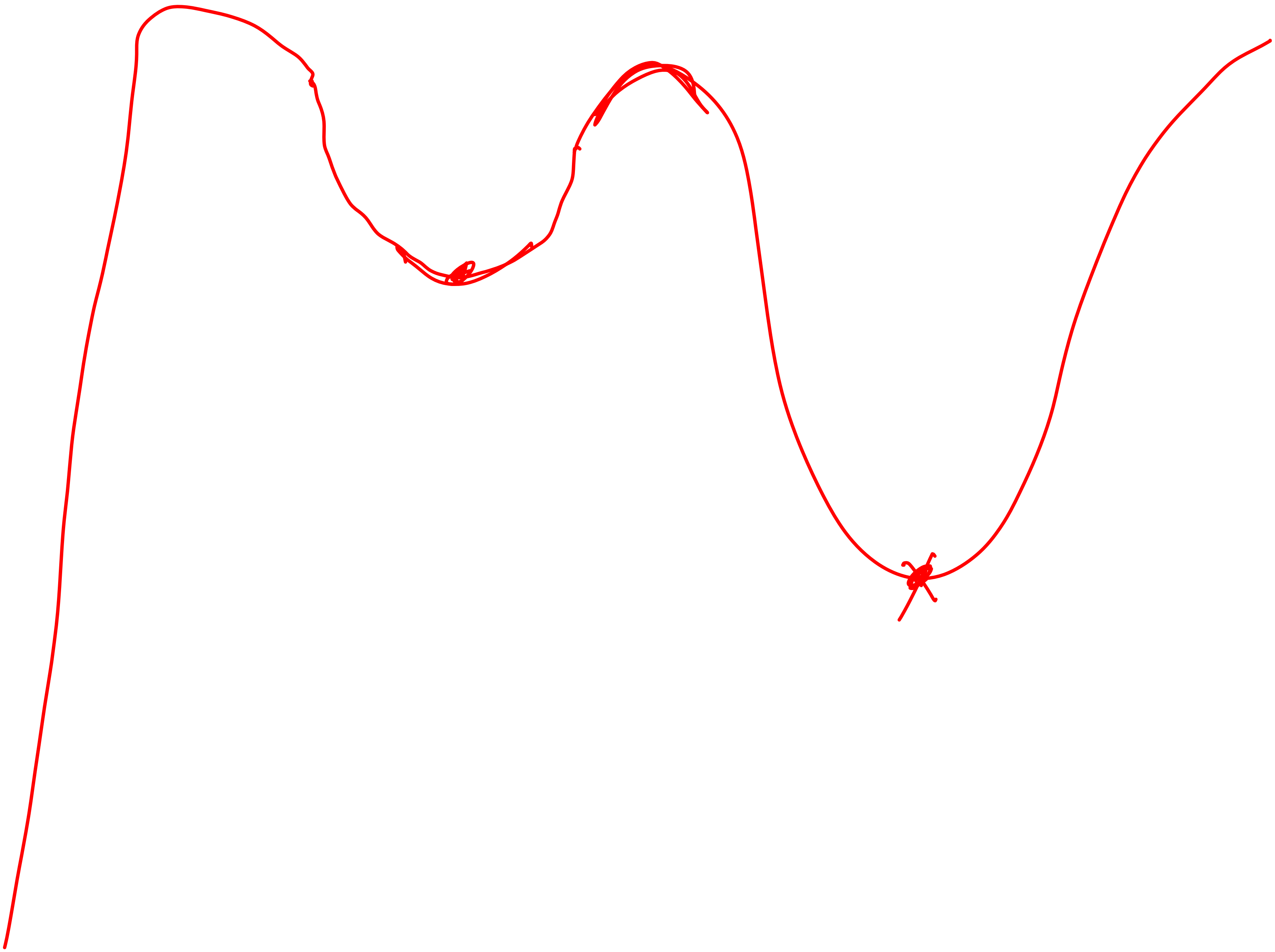
LOCAL MAX AT -1

GRAPH OF f'

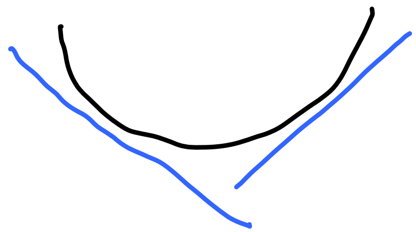


GRAPH OF f





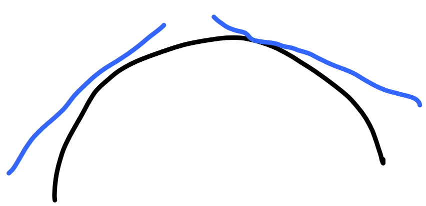
CONCAVITY



CONCAVE UP IF TANGENT LINES BELOW CURVE

NOTE: f' is INCREASING

SO $f'' > 0$



CONCAVE DOWN IF TANGENT LINES ABOVE CURVE.

NOTE: f' IS DECREASING

SO $f'' < 0$

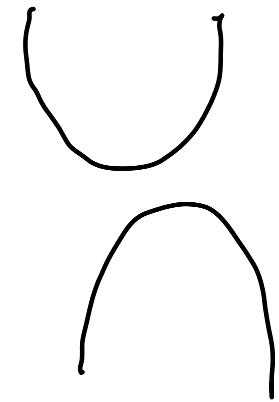
2ND DERIV. TEST:

IF $f'' > 0$ ON AN INTERVAL I THEN THE GRAPH OF f IS CONCAVE UP ON I

IF $f'' < 0$ ON AN INTERVAL I THEN THE GRAPH OF f IS CONCAVE DOWN ON I

WAY TO REMEMBER

$f(x) = x^2 \quad f'(x) = 2x \quad f''(x) = 2 > 0$



$f(x) = -x^2 \quad f'(x) = -2x \quad f''(x) = -2 < 0$

- IF $f''(c) = 0$ and f'' CHANGES SIGN AT C THEN C is CALLED AN INFLECTION POINT

Cor: IF $f'(c) = 0$ AND $f''(c) > 0$ THEN LOCAL MIN AT C

IF $f'(c) = 0$ AND $f''(c) < 0$ THEN LOCAL MAX AT C.

NO INFO ABOUT MAX/MIN IF $f''(c) = 0$.

prev. example

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x^3 - 12x^2 - 24x$

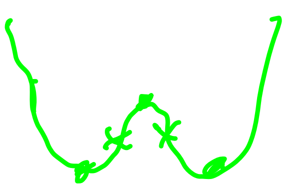
$f''(x) = 36x^2 - 24x - 24$

c.n at 0, 2, -1

$f''(0) = -24$ LOCAL MAX

$f''(2) > 0$ LOCAL MIN

$f''(-1) > 0$ LOCAL MIN



prev. example

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$$

$$f'(x) = 12x^3 - 12x^2 - 24x$$

$$f''(x) = 36x^2 - 24x - 24$$

c.n at $0, 2, -1$

$f''(0) = -24$ LOCAL MAX

$f''(2) > 0$ LOCAL MIN

$f''(-1) > 0$ LOCAL MIN

Inflection points:

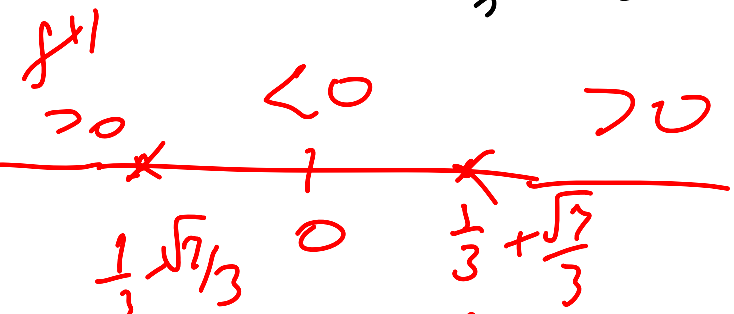
WHERE f'' CHANGES SIGN

$f'' = 0$

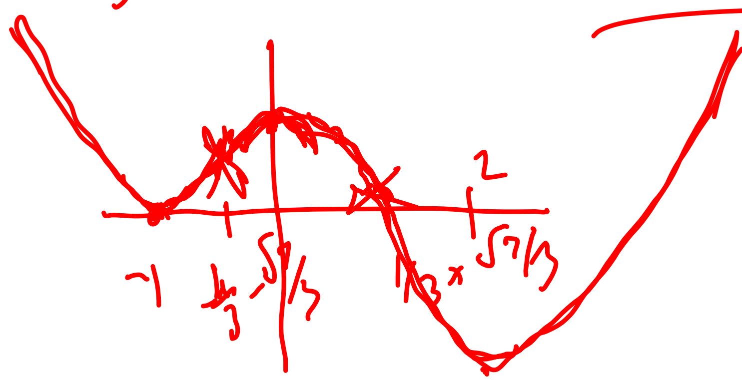
$$f''(x) = 12(3x^2 - 2x - 2)$$

$$x = \frac{2 \pm \sqrt{4+24}}{6} = \frac{2 \pm 2\sqrt{7}}{6} = \frac{1}{3} \pm \frac{\sqrt{7}}{3}$$

INFL. POINTS



GRAPH:



$f'(x) = (x+1)^4 (x-3)^3 (x+2)$

C.N $\quad -1, 3, -2$

NOT LOCAL MAX OR MIN AT -1