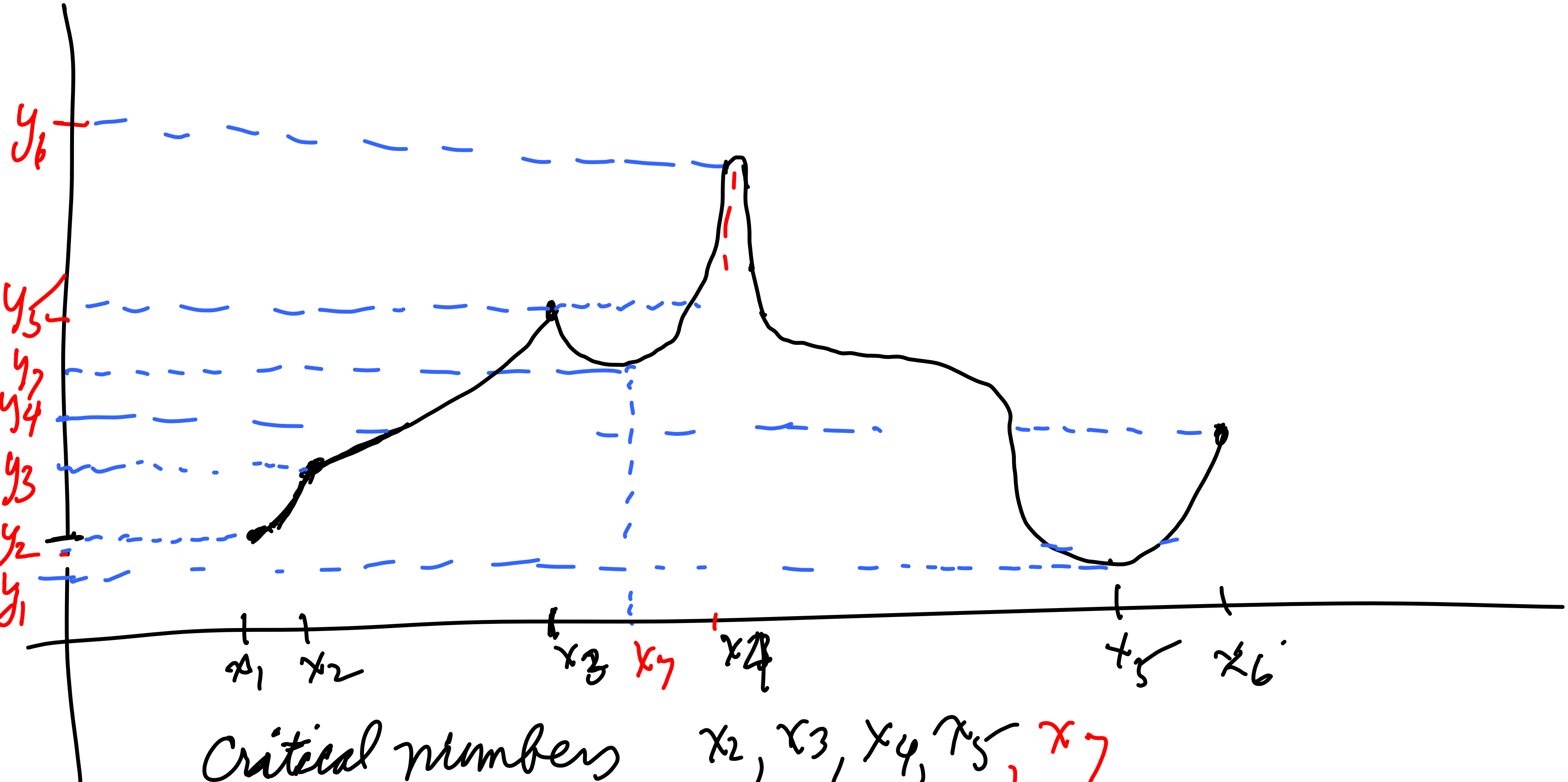
11/9/16



Critical numbers x_2, x_3, x_4, x_5, x_7

endpoints x_1, x_6

critical values y_1, y_3, y_5, y_6, y_7

local max y_5, y_6

local min y_1, y_7

abs. max y_6

abs. min y_1

SIMPLE EXAMPLE

FIND THE (ABSOLUTE) MAX & MIN OF $f(x) = x^3 - 3x^2 + 1$ on $[-\frac{1}{2}, 4]$

$f(4) = 64 - 48 + 1 = 17$

$f(-\frac{1}{2}) = -\frac{1}{8} - \frac{3}{4} + 1 = 1/8$

CRIT. NUMBERS

$f'(x) = 3x^2 - 6x = 0$

$x = 0 \qquad 3x = 6$

$x = 2$

CRIT. VALUES

$f(0) = 1$

$f(2) = 8 - 12 + 1 = -3$

ABS MAX IS EITHER VALUE AT ENDPOINT OR CRITICAL VALUE

$\to$ 17

ABS MIN -3

FIND ABSOLUTE MAX & MIN OF

$$y = 3x - \ln|x| \quad \text{on} \quad \left[\tfrac{1}{4}, 2\right]$$

$\longrightarrow$ $y\left(\frac{1}{4}\right) = 3/4 - \ln\frac{1}{4} = 3/4 + \ln 4 \sim \underline{2.14\ldots}$

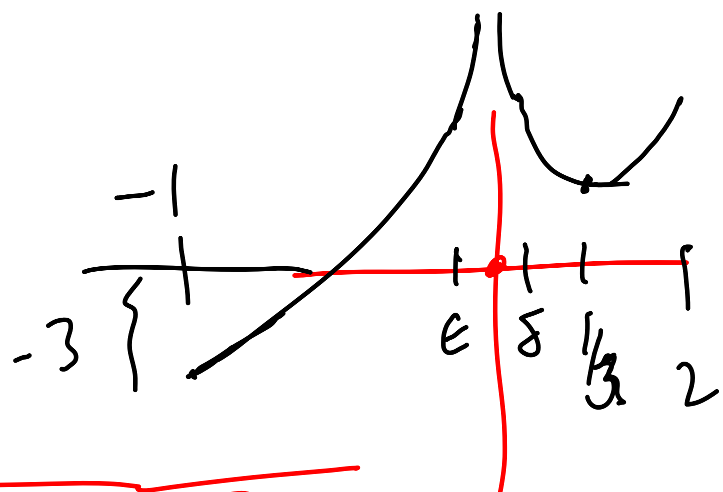
$\longrightarrow$ $y(2) = 6 - \ln 2 \sim \underline{5.31}$

$y' = 3 - \frac{1}{x} = 0 \qquad x = 1/3$ (CRIT. NUMBER)

$\longrightarrow$ $y\left(\frac{1}{3}\right) = 1 - \ln\left(\frac{1}{3}\right) = 1 + \ln 3 \sim \underline{2.10}$

ABS. MAX $\quad 6 - \ln 2$

ABS. MIN $\quad 1 + \ln 3$



MAX MIN ON $[-1, 2]$: $\qquad y(-1) = -3 - \ln 1 = -3$

$y \to +\infty$ as $x \to 0$

NO ABS. MAX.

FIND MIN ON $[-1, \epsilon]$ $\leftarrow$ ϵ small < 0.

NO CRIT. NOS, MIN ON is -3

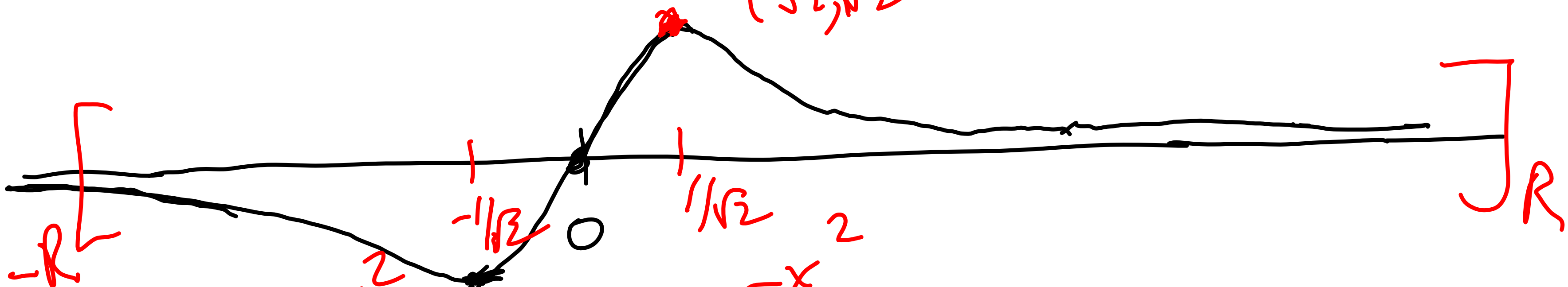
ON $[\delta, 2]$

MIN $= 1 + \ln 3$

Find max of $xe^{-x^2} = y$

$\lim_{x\to-\infty} xe^{-x^2} = \lim_{x\to\infty} xe^{-x^2} = 0$ (WE WILL PROVE THIS NEXT WEEK)

$\left(\frac{1}{\sqrt2}, \frac{1}{\sqrt2}e^{-1/2}\right)$



$y' = e^{-x^2} + x(-2x)e^{-x^2} = 0$

$1 - 2x^2 = 0$

$x^2 = 1/2 \qquad x = \frac{1}{\sqrt2}, -1/\sqrt2$ ← crit numbers

CR. VALUES: $\frac{1}{\sqrt2}e^{-1/2}, -\frac{1}{\sqrt2}e^{-1/2}$

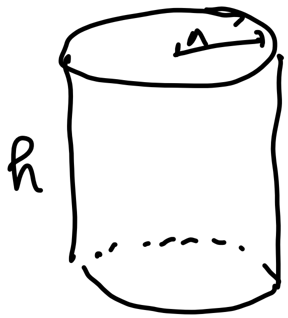
ABS MAX $= \frac{1}{\sqrt2}e^{-1/2}$

ABS MIN $= -\frac{1}{\sqrt2}e^{-1/2}$

$f(-R)$; $f(R)$ very small

ABS MAX, MIN DO NOT OCCUR AT END POINTS

LET $R\to\infty$.



Can: hold $1 L = 1000 cm^3$

FIND CHEAPEST CAN.

MAKE SURFACE AREA AS SMALL AS POSSIBLE

$$A = \underbrace{\pi r^2}_{\text{top}} + \underbrace{\pi r^2}_{\text{bottom}} + 2\pi r h$$

$$1000 \text{ cm} = Vol = \pi r^2 h$$

$$h = \frac{1000}{\pi r^2}$$

$$A = 2\pi r^2 + 2\pi r \left[\frac{1000}{\pi r^2}\right] = 2\pi r^2 + \frac{2000}{r}$$

$\infty > r > 0$

$$A' = 4\pi r - \frac{2000}{r^2} = 0$$

$$4\pi r^3 = 2000$$

$$r^3 = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}}$$

$$h = \frac{1000}{\pi \left[\frac{500}{\pi}\right]^{2/3}}$$