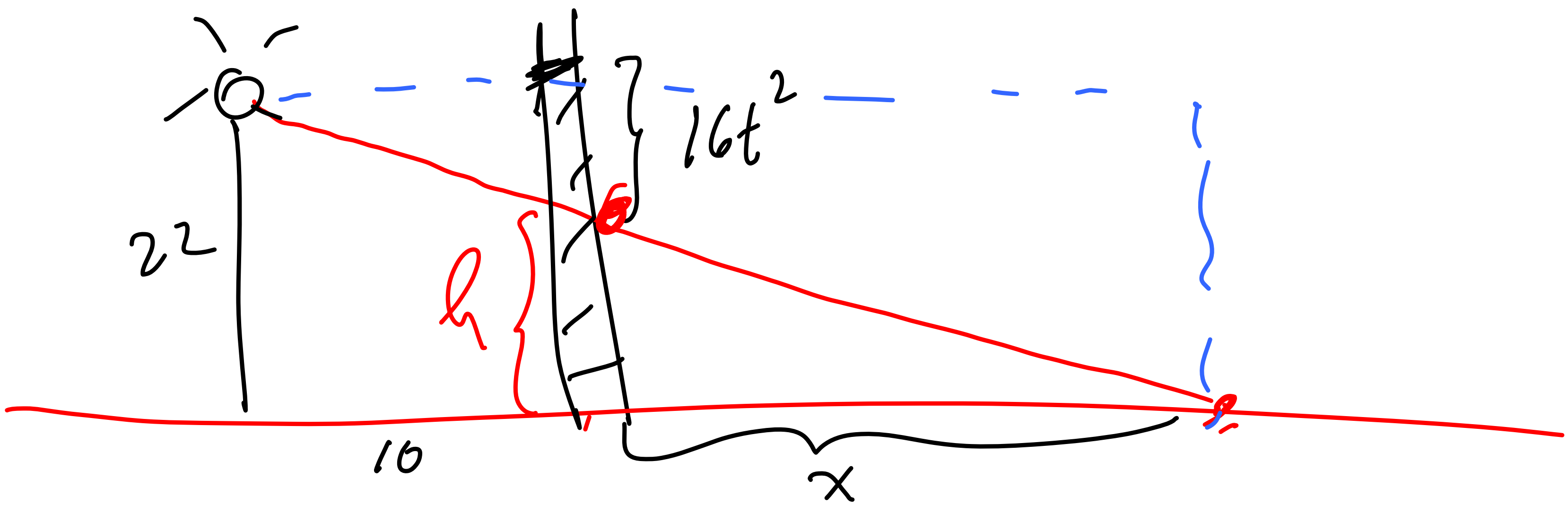
FROM LAST TIME



Find $\frac{dx}{dt}$ when $t = 1/2$.

$h = 22 - 16t^2$

$\frac{dh}{dt} = -32t$

$\frac{h}{x} = \frac{22}{10+x}$

or $\frac{h}{x} = \frac{16t^2}{10}$

$\frac{22-16t^2}{x} = \frac{22}{10+x}$

$220 + 22x - 160t^2 - 16t^2 x$

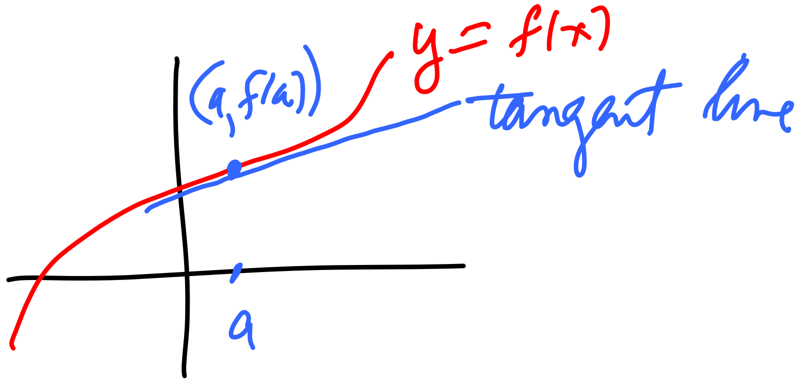
$= 22x$

$220 - 160t^2 = 16t^2 x$

$220/16t^2 - 10 = x$

OTHER WAYS

3.10 Linear Approx. & Differentials



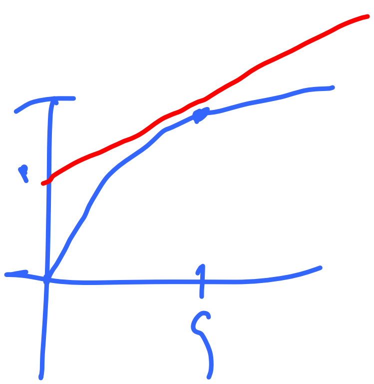
$$f(x) \approx f(a) + f'(a)(x-a)$$

RECALL: TO ANSWER QUESTIONS ABOUT A COMPLICATED f near a USE THE LINEAR FUNCTION

$$L(x) = f(a) + f'(a)(x-a)$$

EXAMPLE $\sqrt{9.01} \approx ?$

$f(x) = \sqrt{x}$ $\quad a = 9 \quad f(a) = 3 \quad f'(a) = 1/6$



$$L(x) = 3 + \frac{1}{6}(x-9)$$

$$L(9.01) = 3 + \frac{1}{6}(.01) \approx 3.00166\ldots$$

USEFUL:

$$\underbrace{f(x)-f(a)}_{\text{CHANGE IN } f} \simeq f'(a)\underbrace{(x-a)}_{\text{CHANGE IN } x}$$

CHANGE 9 to 9.01 SQUARE ROOT CHANGES BY $\approx \frac{1}{2\sqrt{9}}(.01)$

a x

OTHER NOTATION:

$$\underbrace{f(x+\Delta x)-f(x)}_{\text{CHANGE IN } f} \approx f'(x)\underbrace{\Delta x}_{\text{CHANGE IN } x}$$

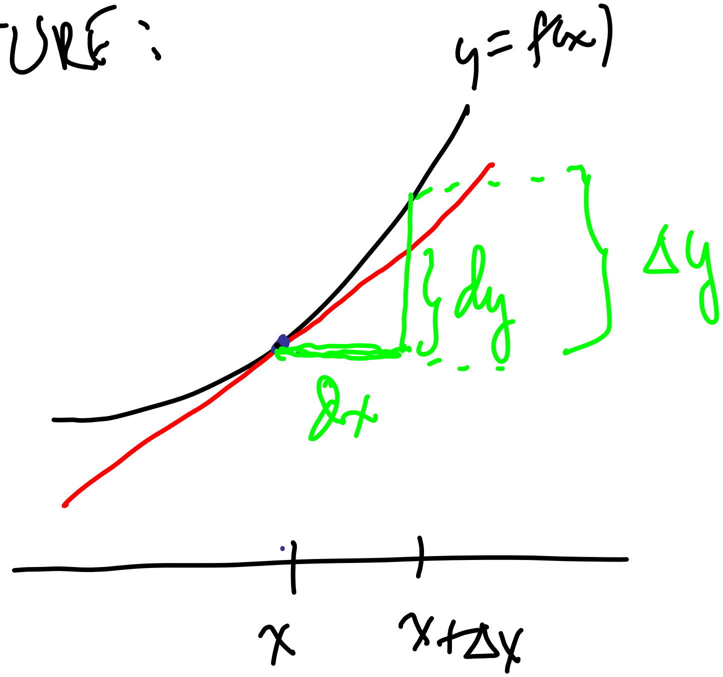
(THINK OF x AS FIXED, Δx CHANGING)

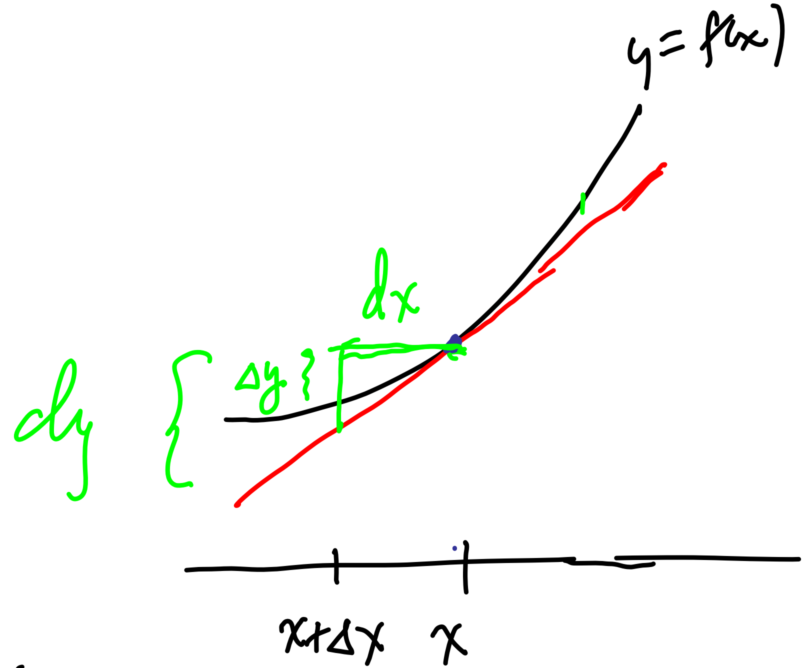
if $y=f(x)$ the CHANGE IN y.

$$\underbrace{\Delta y}_{\text{CHANGE IN } y} \approx \underbrace{f'(x)\Delta x}_{\text{CHANGE IN LINEARIZATION}}$$

Differentials: $dy = f'(x)\,dx$ change in linearization NOTE

PICTURE:





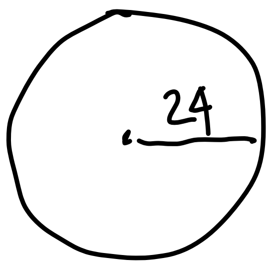
Here $dy < \Delta y$ true change

change in linear approx.

If f' is increasing then $dy < \Delta y$

↳ i.e. $f'' > 0$ [SO LINEAR APPROX IS SMALLER THAN TRUE FUNCTION]

If $f''(x) < 0$ then $\Delta y < dy$



$$\text{Area} = \pi(24)^2$$

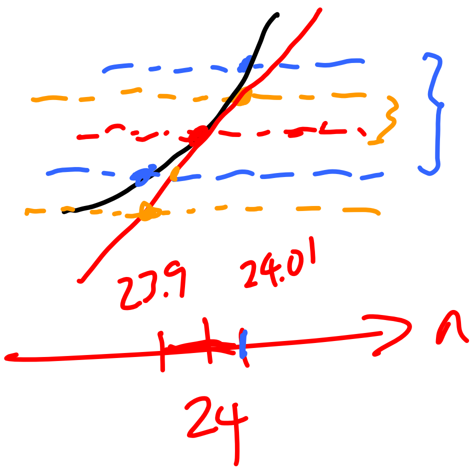
Suppose radius has possible measurement error of 0.1

Use differentials to approx. error in Area

$$A = \pi r^2$$

$$dA = 2\pi r\, dr$$

$$dA = 2\pi(24)(0.1)$$



IS THE TRUE ERROR ΔA SMALLER OR LARGER?

Relative error: $\dfrac{dA}{A} = \dfrac{2\pi r\, dr}{\pi r^2} = \dfrac{2}{r}\, dr$

$$= \frac{2}{24}(.1)$$

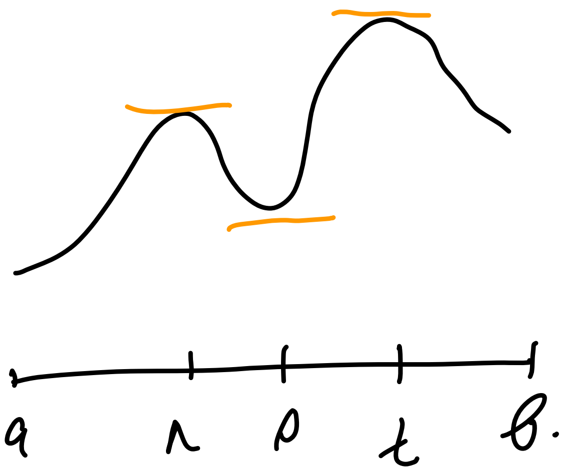
KEY IN ALL OF THESE PROBLEMS:

APPROXIMATE A COMPLICATED FUNCTION $f(x)$

BY THE LINEAR FUNCTION $f(a) + f'(a)(x-a)$

READ 4.1 FOR WEDNESDAY FRI IS HOLIDAY

4.1 SUPPOSE f DEFINED ON AN INTERVAL I.



ABSOLUTE MAX $f(c)$

MEANS: $f(x) \le f(c)$ ALL x in I

WORDING: ABS MAX OCCURS AT C. THE ABS MAX IS $f(c)$

ABSOLUTE MIN AT C

$f(x) \geq f(c)$

LOCAL MAX AT C

$f(x) \le f(c)$ ALL x NEAR C

LOCAL MIN AT C

$f(x) \geq f(c)$ ALL x NEAR C

- "NEAR" MEANS ON OPEN INTERVAL $(c-d, c+d)$ for some $d > 0$

ABS MAX $f(t)$

ABS MIN $f(a)$

LOCAL MAX $f(r)$, $f(t)$

LOCAL MIN $f(p)$

NOT AT ENDPOINTS

EXTREME VALUE THEOREM

IF f IS CONTINUOUS ON $[a,b]$ THEN f HAS AN ABS MAX AND AN ABS MIN.

FERMAT: IF f HAS A LOCAL MAX OR LOCAL MIN AT c AND IF $f'(c)$ EXISTS, THEN $f'(c)=0$.

DEF: A CRITICAL NUMBER IS A NUMBER c IN DOMAIN OF f SO THAT $f'(c)=0$ OR $f'(c)$ DOES NOT EXIST.
$f(c)$ IS CALLED A CRITICAL VALUE.

ABS MIN, ABS MAX OCCUR AT CRITICAL POINTS OR END POINTS.

KEY POINT TO FIND ABS MAX:
LIST CRITICAL VALUES & VALUES AT ENDPOINTS, THEN TAKE MAX

USUALLY LIST IS SMALL