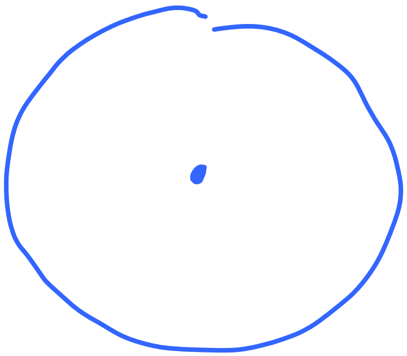
RELATED RATES (SECTION 3.9) 11/3/14



oil spill 50 cu ft per min $\frac{1}{10}$" THICK

HOW FAST IS RADIUS CHANGING
WHEN RADIUS = 30 FT. ?

$V(t)$ = vol at time t
$r(t)$ = radius at time t

$$V = \pi r^2 \cdot \left[\frac{1}{10} \text{ in}\right]\left[\frac{1 \text{ ft}}{12 \text{ in}}\right] \quad \text{cu ft.}$$

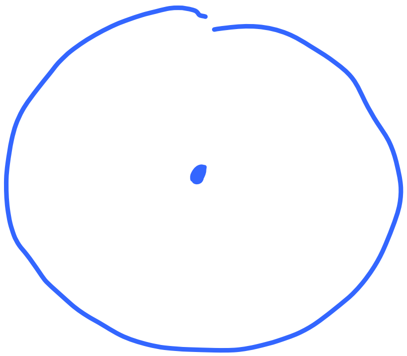
$$\frac{dV}{dt} = 50 \qquad \text{want } \frac{dr}{dt} \qquad \text{when } r = 30$$

$$\frac{dV}{dt} = \frac{\pi}{120}\left[2r\frac{dr}{dt}\right] \qquad 50 = \frac{\pi}{120} \cdot 2(30)\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{100}{\pi} \text{ ft/min}$$

RELATED RATES

New problem



oil spill $\frac{1}{10}$'' thick

when radius = 50 feet

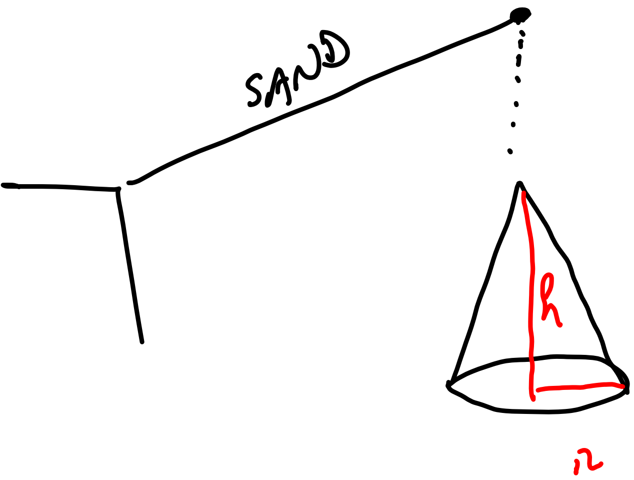
measure $\frac{dr}{dt} = 25$ ft/min

At what rate is the oil being spilled

(cu ft/min)

$$V = \pi r^2 \cdot \frac{1}{120}$$

$$\frac{dV}{dt} = \frac{2\pi r}{120}\frac{dr}{dt} = \frac{2\pi}{120}\cdot 50 \cdot 25$$



height = h

radius = r

— SUPPOSE $h = 2r$

— 30 cu.ft PER MIN

HOW FAST IS HEIGHT CHANGING WHEN $h = 20$?

$$V = \frac{\pi r^2 h}{3} \qquad r = h/2$$

$$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 \cdot h = \frac{\pi h^3}{12}$$

$$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt}$$

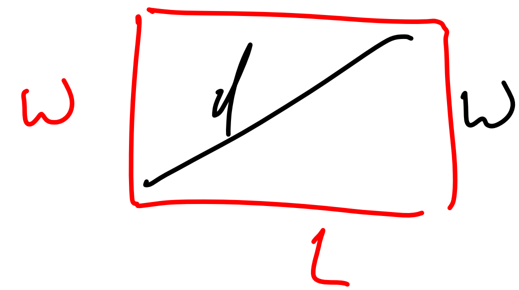
$$30 = \frac{\pi}{4}(20)^2 \frac{dh}{dt} \qquad \frac{dh}{dt} = \frac{120}{\pi \cdot 400}$$

Sp. 2011 #4

RECTANGLE

LENGTH INCREASES 3 FEET/MIN

WIDTH DECREASES 2 FEET/MIN



WHEN LENGTH = 15 & WIDTH = 8:

a. HOW FAST IS THE AREA CHANGING?

$$A = L \cdot W \qquad \frac{dA}{dt} = \frac{dL}{dt} \cdot W + L \cdot \frac{dW}{dt} = 3 \cdot 8 + 15(-2)$$

$$= -6 \text{ sq ft/min}$$

b. HOW FAST IS THE PERIMETER CHANGING?

$$P = 2W + 2L \qquad \frac{dP}{dt} = 2\frac{dW}{dt} + 2\frac{dL}{dt} = 2(-2) + 2(3)$$

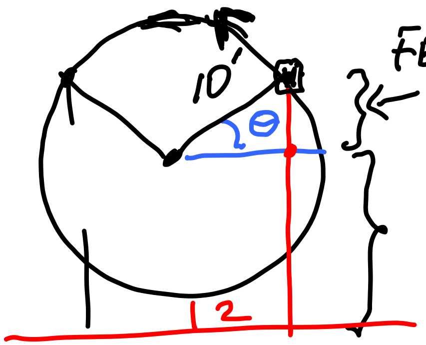
$$= 2 \text{ ft/min}$$

c. HOW FAST IS THE DIAGONAL CHANGING?

$$D = \sqrt{L^2 + W^2}$$

$$\frac{dD}{dt} = \frac{1}{2}\left(L^2 + W^2\right)^{-1/2}\left[2L\frac{dL}{dt} + 2W\frac{dW}{dt}\right]$$

$$= \frac{1}{2}\left(15^2 + 8^2\right)^{-1/2}\left[2 \cdot 15 \cdot 3 + 2 \cdot 8(-2)\right]$$



FERRIS WHEEL 1 REV. EVERY 2 MINUTES

HOW FAST IS RIDER RISING WHEN 18' ABOVE GROUND?

$$\frac{d\theta}{dt} = \frac{2\pi \text{ rad}}{2 \text{ min}} = \pi \text{ rad/min}$$

$18-12$

$$6 = 10 \sin\theta$$

$$\frac{3}{5} = \sin\theta$$

$$h = 12 + 10\sin\theta$$

$$\cos\theta = \pm\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$= \pm\sqrt{1-\frac{9}{25}}$$

$$= \pm 4/5$$

$$\frac{dh}{dt} = 10\left[\cos\theta\right]\frac{d\theta}{dt}$$

$$\frac{dh}{dt} = 10 \cdot \frac{4}{5} \cdot \pi \text{ ft/min.}$$

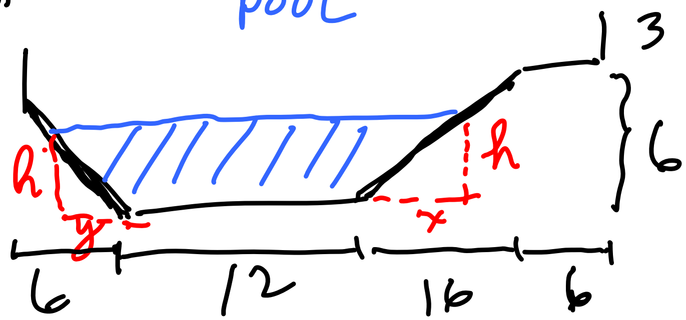
$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

p. 249 # 26

POOL



20 ft WIDE

FILL @ $0.8\ ft^3/min$

HOW FAST IS WATER RISING WHEN 5 FEET DEEP?

Vol = CROSS SECTIONAL AREA × WIDTH.

$$\frac{h}{x} = \frac{6}{16}$$

$$\frac{h}{y} = \frac{6}{6}$$

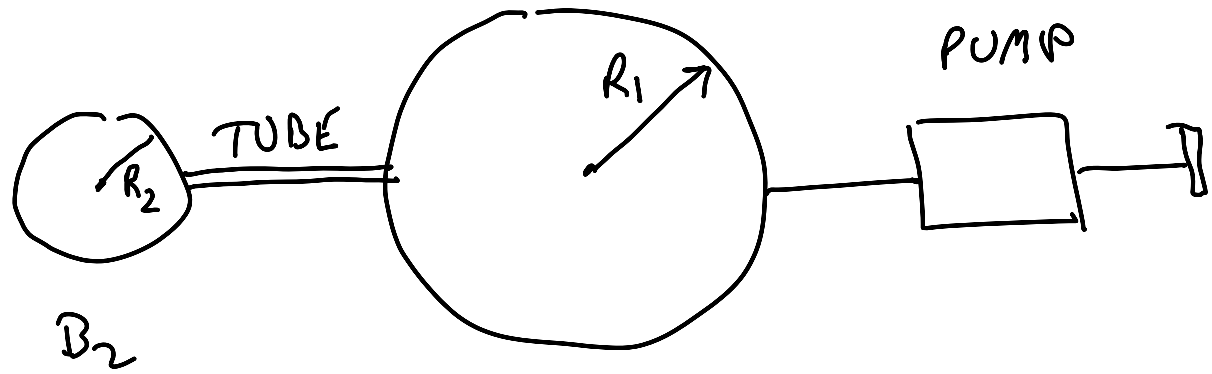
$$h = y$$

$$h = \frac{3}{8}x$$

$$V = 20\left[(h)(12 + x + y) - \frac{1}{2}hy - \frac{1}{2}hx\right]$$

$$V = 20\left[h\left(12 + \frac{8}{3}h + h\right) - \frac{1}{2}h \cdot h - \frac{1}{2}h\left(\frac{8}{3}h\right)\right]$$

WIN 2011 #6



AIR PUMPED IN @ 101 CM^3/MIN; LEAKS OUT (INTO B_2) @ rate $\pi \frac{dR_1}{dt} \frac{cm^3}{MIN}$

AT TIME t_0 $R_1 = 5$ $R_2 = 2$. HOW FAST is R_2 CHANGING?

Volume of a sphere $= \frac{4}{3}\pi R^3$

$$V_1 = \frac{4}{3}\pi R_1^3$$

$$\frac{dV_1}{dt} = \frac{4}{3}\pi \cdot 3R_1^2 \frac{dR_1}{dt}$$

$$\frac{dV_1}{dt} = 101 - \frac{\pi dR_1}{dt}$$

$$V_2 = \frac{4}{3}\pi R_2^3$$

$$4\pi R_1^2 \frac{dR_1}{dt} = 101 - \pi \frac{dR_1}{dt}$$

$$\frac{\pi dR_1}{dt} = \frac{dV_2}{dt} = 4\pi R_2^2 \frac{dR_2}{dt}$$

$$4\pi \cdot 25 \frac{dR_1}{dt} = 101 - \pi \frac{dR_1}{dt}$$

$$(100\pi + \pi)\frac{dR_1}{dt} = 101$$

$$\frac{dR_1}{dt} = \frac{101}{101\pi} = \frac{1}{\pi}$$

$$\pi\left(\frac{1}{\pi}\right) = 4\pi \cdot 4 \frac{dR_2}{dt}$$

$$\boxed{\frac{1}{16\pi} = \frac{dR_2}{dt}}$$