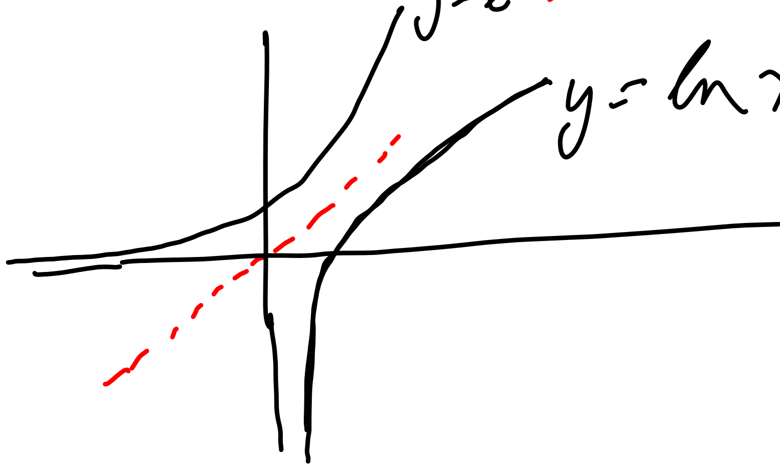
10/31

Last time: $\ln x$ defined for $x>0$



(defined only for $x>0$)

$$\frac{d}{dx}\ln x = \frac{1}{x} \quad (x>0)$$

$$\ln|x| = \begin{cases} \ln x & \text{if } x>0 \\ \ln(-x) & \text{if } x<0 \end{cases}$$

$$\frac{d f^{-1}}{dx}(x) = \frac{1}{f'(f^{-1}(x))}$$

$$\frac{d}{dx}\ln|x| = \begin{cases} \frac{1}{x} & \text{if } x>0 \\ \left(\frac{1}{-x}\right)(-1) & \end{cases}$$

so $\frac{d}{dx}\ln|x| = \frac{1}{x}$

$(x \neq 0)$

$$[f(g(x))]' = f'(g(x))\,g'(x)$$

$$y = \ln|\cos x|$$

$$y' = \frac{1}{\cos x} \cdot (-\sin x)$$

IMPLICIT FUNCTIONS

$$x^3 + y^3 = \frac{9}{2} x y$$

$(1,2)$ on curve (check)

What does the curve look like near $(1,2)$?

$\boxed{\text{Suppose } y \text{ is a function of } x \text{ near } (1,2)}$

$$x^3 + y(x)^3 = \frac{9}{2} x \, y(x)$$

$$3x^2 + 3\left[y(x)\right]^2 y'(x) = \frac{9}{2}\left[1 \cdot y(x) + x \, y'(x)\right]$$

at $(1,2)$

$$3 \cdot 1 + 3(2)^2 y'(1) = \frac{9}{2}\left[2 + 1 \cdot y'(1)\right]$$

Solve for $y'(1)$

$$3 + 12 y'(1) = 9 + \frac{9}{2} y'(1)$$

$$\left(12 - \frac{9}{2}\right) y'(1) = 6 \qquad y'(1) = \frac{6}{15/2} = \frac{12}{15} = \frac{4}{5}$$

Tangent line at $(1,2)$ is $y = \frac{4}{5}(x-1) + 2$

Properties of ln:

$$\ln(ab) = \ln a + \ln b \qquad \ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

$$\ln(a^b) = b \ln a. \qquad \ln(a^{-b}) = -b \ln a$$

$$y = \ln\left((x-4)^2 (2x-5)^3\right) = \ln\left[(x-4)^2\right] + \ln\left[(2x-5)^3\right]$$

$$= 2\ln(x-4) + 3\ln(2x-5)$$

$$y' = 2\frac{1}{x-4} + 3\frac{1}{2x-5}\cdot 2$$

y

$$y = \frac{(3x-4)^5 (x-5)^2 (x+1)^3}{8x^3 (7x-5)^2}$$

$$\ln|y| = 5\ln|3x-4| + 2\ln|x-5| + 3\ln|x+1| - \ln 8 - 3\ln|x| - 2\ln|7x-5|$$

y is a function of x.
Use chain rule:

$$\frac{y'(x)}{y(x)} = \frac{5\cdot 3}{3x-4} + \frac{2}{x-5} + \frac{3}{x+1} - \frac{3}{x} - \frac{2\cdot 7}{7x-5}$$

$$y' = \left[\frac{(3x-4)^5 (x-5)^2 (x+1)^3}{8x^3 (7x-5)^2}\right]\left[\frac{15}{3x-4} + \frac{2}{x-5} + \frac{3}{x+1} - \frac{3}{x} - \frac{14}{7x-5}\right]$$

$$\frac{d}{dx} x^p = p x^{p-1}$$

$e^{\pi/\pi e}$ $\pi^{e/\pi e}$ $e^{\pi} > \pi^e$

$e^{1/e}$ $\pi^{1/\pi}$

$$\frac{d}{dx} p^x = \frac{d}{dx}\left[e^{\ln p}\right]^x = \frac{d}{dx} e^{(\ln p)x} = e^{(\ln p)\cdot x} \ln p$$

$$= p^x \ln p$$

$y = p^x$ $\quad \ln|y| = x \ln p$

$\frac{y'}{y} = \ln p$ $\quad y' = y \ln p = p^x \cdot \ln p$

$x > 0$

$$\frac{d}{dx} x^{\frac{1}{x}}$$

$y = x^{\frac{1}{x}}$ $\quad \ln|y| = \frac{1}{x} \ln|x|$

$$\frac{y'}{y} = \frac{x \cdot \frac{1}{x} - \ln|x| \cdot 1}{x^2} \qquad y' = x^{\frac{1}{x}}\left[\frac{1 - \ln|x|}{x^2}\right]$$

1. $y = \sqrt{x + \sin(x + \sqrt{x})}$

2. $y = (1+x)^{\cos x}$

3. $y = \arctan\left(\frac{1}{2+x^2}\right)$

4. $y = \pi^\pi + x^\pi + \pi^x + x^x$

5. $y = \arctan\left(\frac{t+6}{1-6t}\right)$

6. $y = \cos(xe^{-x^2} + 1)$

7. $y = (4x^2+1)^{3/2} \arctan(2x^2)$

8. $y = (x^2+1)^{\sin x}$

9. $y = [\sin x]^{\ln x}$

ALL FROM OLD FINAL EXAMS

IF BASE AND EXPONENT ARE VARIABLE USE

$$B^c = e^{(\ln B)c}$$

OR

$$\ln|y| = \cdots$$

$$y = (x^3+1)^{\sin x}$$

$$\ln|y| = \sin x \ln(x^2+1)$$

$$\frac{y'}{y} = (\cos x) \ln(x^2+1) + \sin x \, \frac{1}{x^2+1} \cdot 2x$$

$$y' = (x^2+1)^{\sin x} \left[\cos x \ln(x^2+1) + \frac{(\sin x)(2x)}{x^2+1} \right]$$

W: 3.9