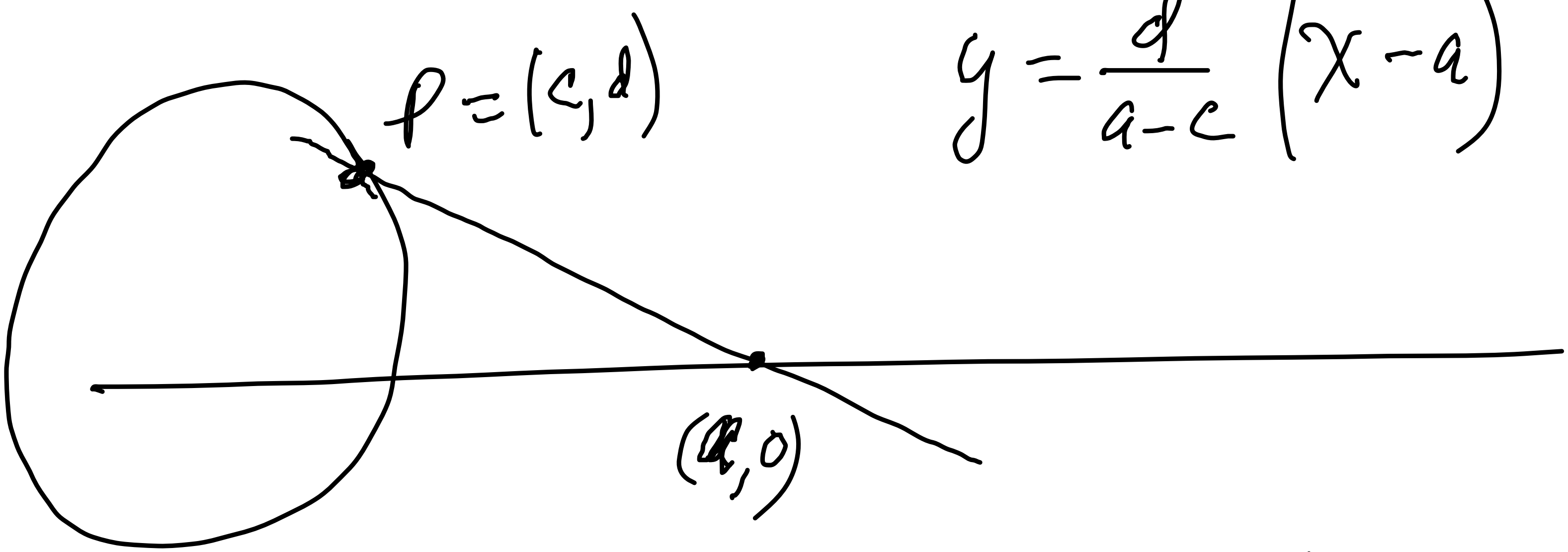
QUESTIONS ON PARAMETRIC EQUATIONS?



$$y = \frac{d}{a-c}(x-a)$$

$\boxed{x(0) = a}$

$y(0) = 0$

$x(t) =$

$y(t) =$

Const rate.

$t = 2.4$

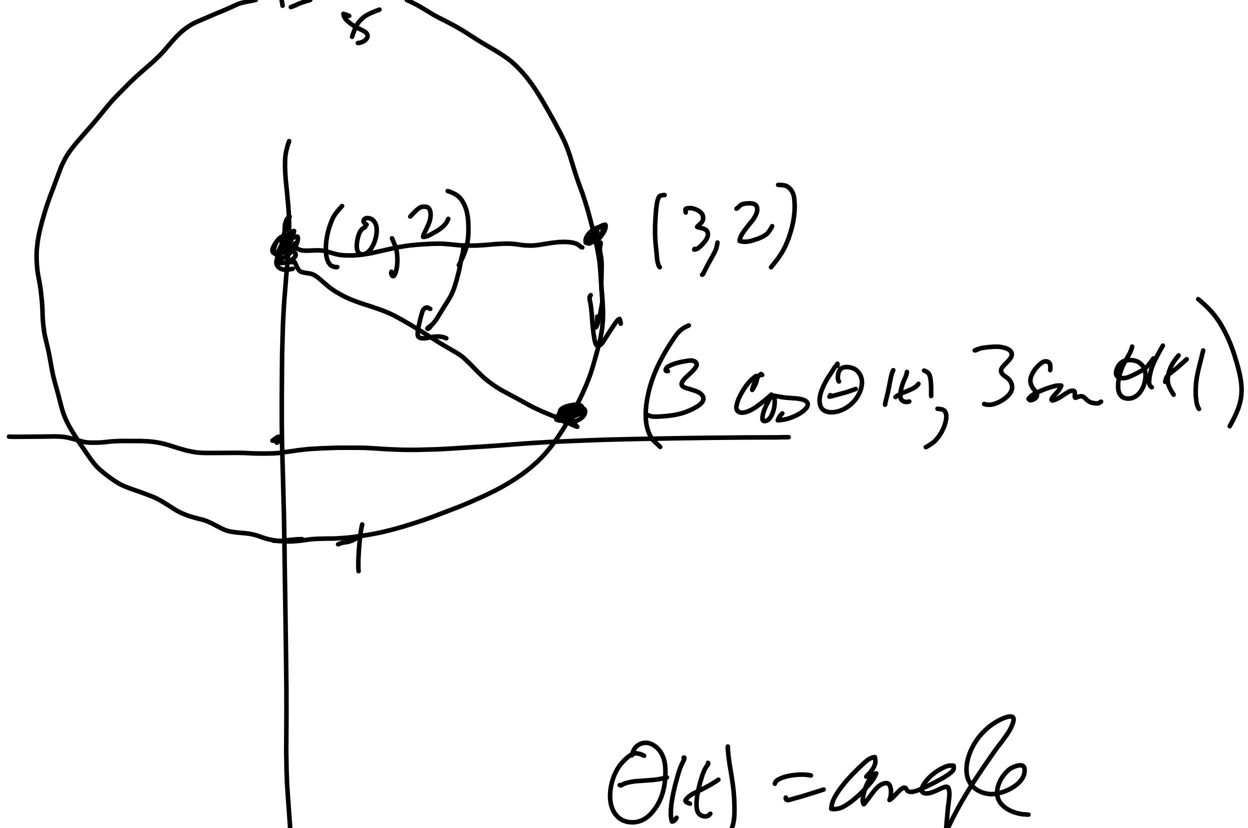
$\boxed{x(2.4) = c}$

$y(2.4) = d$

$x(t) = At + B \qquad B = a$

$$x(t) = \left(\frac{c-a}{2.4}\right)t + a$$

$c = A \cdot 2.4 + a \qquad \frac{c-a}{2.4} = A$



$\theta(t) = $ angle

$0 \le t \le 2\pi$ travel once

$\theta(t) = -t$

$x(t) = 3\cos(-t)$
$y(t) = 3\sin(-t) + 2$

$x_1 = 3\sin t$
$y_1 = 2\cos t$
$x_2 = -3 + \cos t$
$y_2 = 1 + \sin t$

$x_1 = -3$
$y_1 = 0$

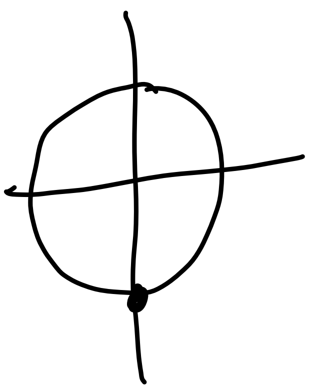
Find t so that $(x_1(t), y_1(t)) = (x_2(t), y_2(t))$

$3\sin t = -3 + \cos t$
$2\cos t = 1 + \sin t \longrightarrow 2\cos t - 1 = \sin t$

$3(2\cos t - 1) = -3 + \cos t$

$4\cos t = 0$
$\cos t = 0$

$-1 = \sin t$



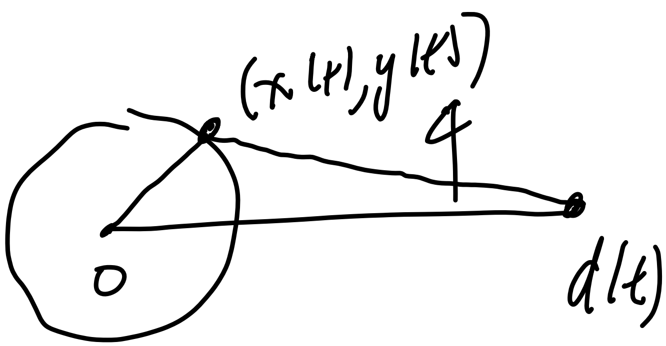
$t = \frac{3\pi}{2} + k \cdot 2\pi$ k integer

$$3.5 \frac{\text{rev}}{\text{sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 7\pi \text{ rad/sec}$$

$$\theta(t) = 7\pi t$$

$$x(t) = 2\cos(7\pi t)$$

$$y(t) = 2\sin(7\pi t)$$



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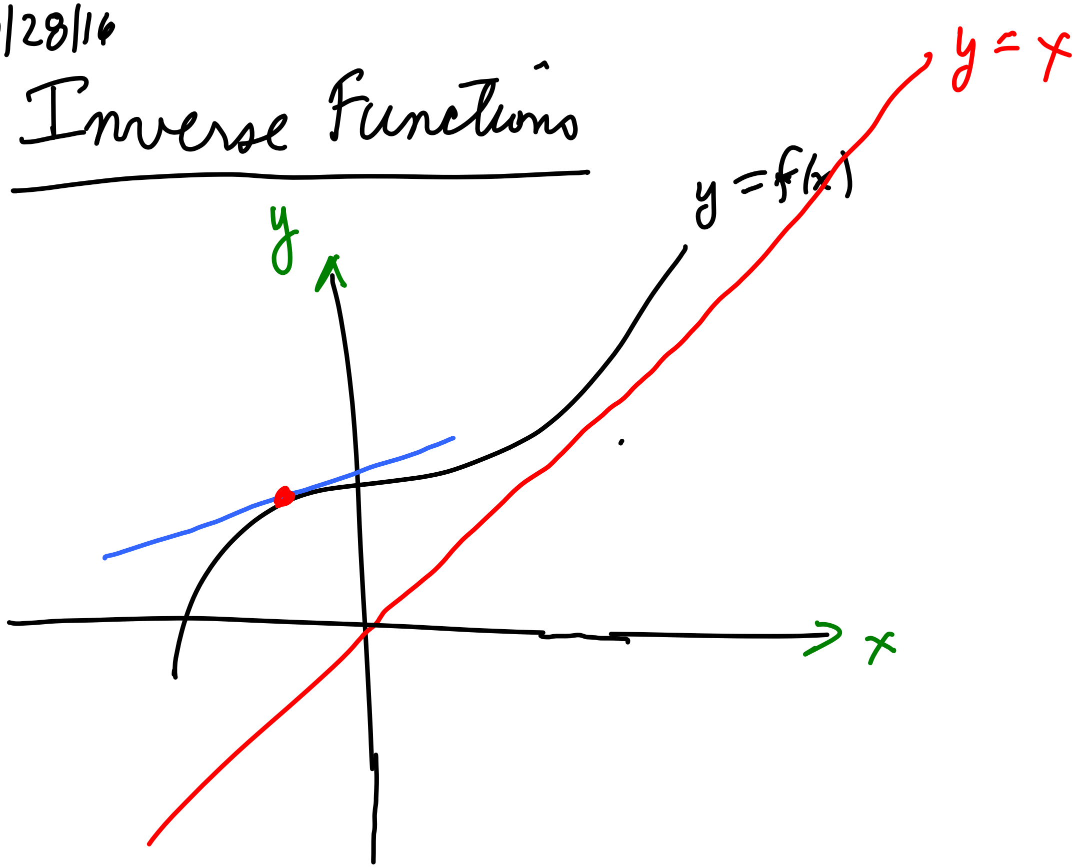
Inverse Functions



$y = f(x)$

$f^{-1}(y) = x$

To get the graph of $y = f^{-1}(x)$ just switch x & y ie reflect about the line $y = x$

But the tangent line also reflect

So if $y = f(x)$ has a tangent line at $(x_1, f(x_1))$ then $y = f^{-1}(x)$ has a tangent line at $(x_1, f^{-1}(x_1))$

Switching x & y then rise/run becomes run/rise, but at a different point.

Suppose $g = f^{-1}$

then $f(g(x)) = x$ (AND $g(f(x)) = x$)

By the chain rule:

$$f'(g(x))\, g'(x) = 1$$

so $g'(x) = \dfrac{1}{f'(g(x))}$

not at x

Reword

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Examples ① $f(x) = e^x$ $\qquad y = e^x > 0$

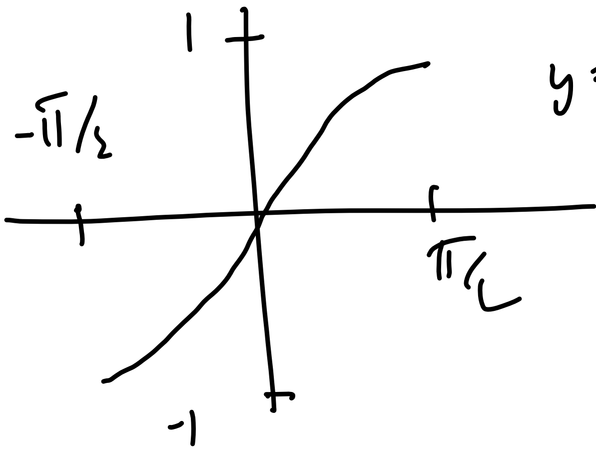
$f^{-1}(x) = \ln x \longleftarrow$ defined for $x > 0$

$$(f^{-1})'(x) = \frac{1}{e^{\ln x}} = \frac{1}{x} \qquad x > 0$$

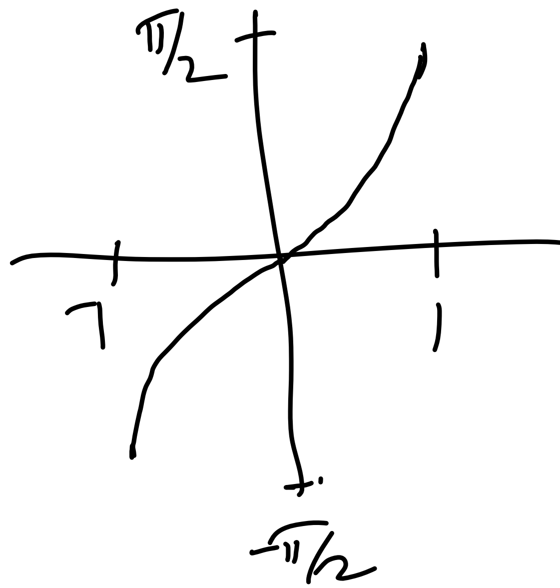
② $$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{\sec^2(\tan^{-1} x)} = \frac{1}{\tan^2(\tan^{-1}(x)) + 1}$$

$$= \frac{1}{x^2+1}$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\cos\left(\sin^{-1}(x)\right)}$$



$y = \sin x$



$y = \sin^{-1}(x)$

$$\cos x = \pm\sqrt{1-\sin^2 x}$$

$+$ because $\cos x > 0$ for $-\pi/2 < x < \pi/2$.

so $\cos\left(\sin^{-1}(x)\right) = \sqrt{1-x^2}$

and $$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$