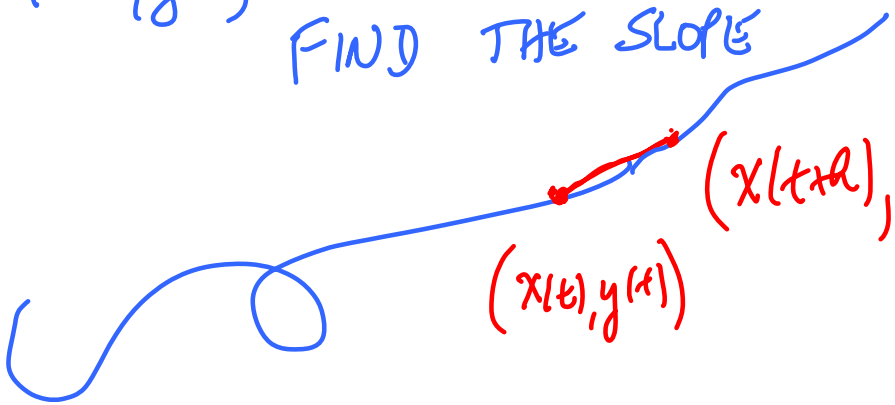
10/26/16

parametric equations and calculus

[10.2 – JUST FIRST PART
SKIP { AREA, ARC LENGTH, SURFACE AREA

$(x(t), y(t))$ PARAMETRIC EQ FOR A CURVE

FIND THE SLOPE



$$\frac{y(t+h)-y(t)}{x(t+h)-x(t)} = \frac{\frac{y(t+h)-y(t)}{h}}{\frac{x(t+h)-x(t)}{h}} \to \frac{\dot{y}(t)}{\dot{x}(t)}$$

(if $x'(t) \neq 0$)

$\dot{y}$ means $\frac{dy}{dt}$

So at t: slope $= \frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$

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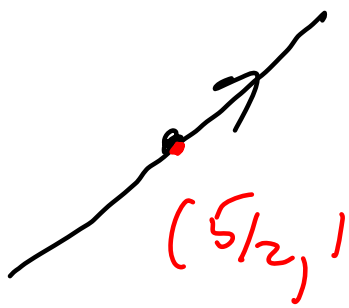
mnemonic. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{dx/dt}$ i.e. $y(t(x))$ ←

$y = t^2 \qquad x = 3t - (t^3)/2$

Find equation of tangent line when $t = 1$

$y(1) = 1 \qquad x(1) = 3 - 1/2 = 5/2$

$$\text{SLOPE} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{3 - \frac{3}{2}t^2} = \frac{2}{3/2} = 4/3$$

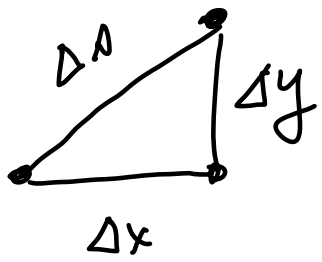


$(5/2, 1)$

$$y = \frac{4}{3}\left(x - 5/2\right) + 1$$

SPEED

change in dist along curve

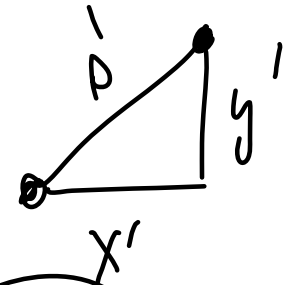


$$\Delta x \approx x'(t)\Delta t$$

$$\Delta y \approx y'(t)\Delta t$$

$$\Delta s \approx s'(t)\Delta t$$

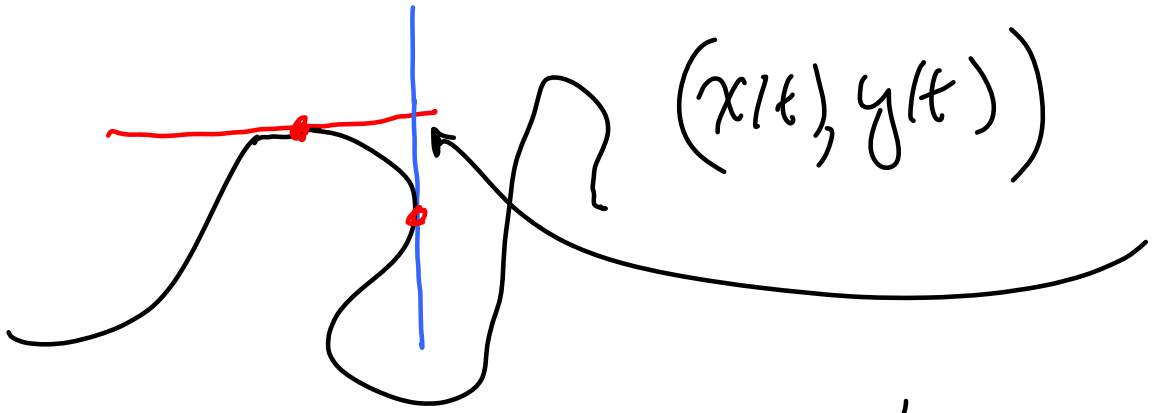
DIVIDE BY Δt



$$s' = \sqrt{(x')^2 + (y')^2}$$

★ (x', y') POINTS IN DIRECTION OF CURVE (TANGENT TO CURVE)

"2D VELOCITY"



$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$$

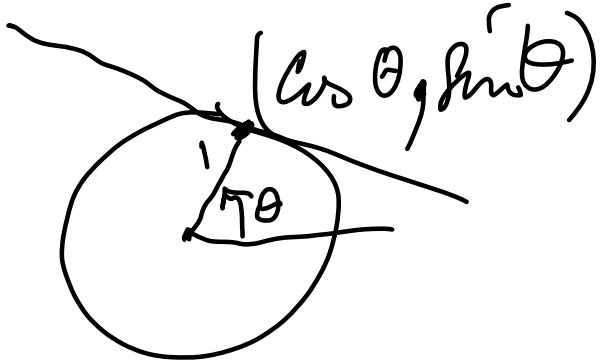
HORIZONTAL: MUST HAVE $y'(t) = 0$

VERTICAL: MUST HAVE $x'(t) = 0$

BUT SOMETIMES $y' = 0$ AND NOT HORIZONTAL

EXAMPLE: $x(t) = t^2$ $\quad y(t) = 3t^2$ Find slope when $t = 0$

$$\frac{dy/dt}{dx/dt} = \frac{6t}{2t} = 3$$



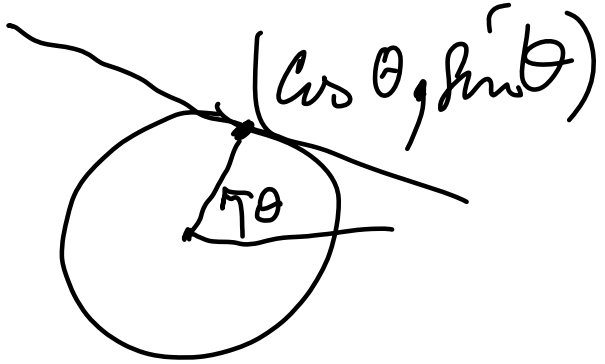
SLOPE ?

$x = \cos\theta \quad y = \sin\theta$

$\dot{x} = -\sin\theta \quad \dot{y} = \cos\theta$

$$\text{slope} = \frac{\dot{y}}{\dot{x}} = \frac{\cos\theta}{-\sin\theta} = -\cot\theta$$

or $\text{slope} = -\frac{x}{y}$ (knew before)



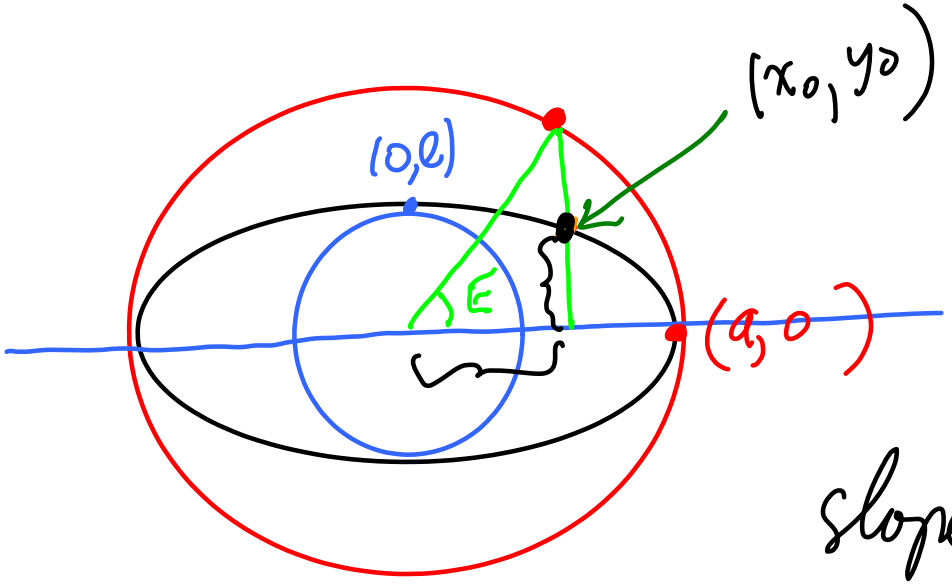
SLOPE ?

$x = \cos\theta \quad y = \sin\theta$

$x' = -\sin\theta \quad y' = \cos\theta$

$$\text{slope} = \frac{y'}{x'} = \frac{\cos\theta}{-\sin\theta} = -\cot\theta$$

$$\Rightarrow \text{slope} = -\frac{x}{y}$$ (Knew Before)



$x(E) = a\cos E$

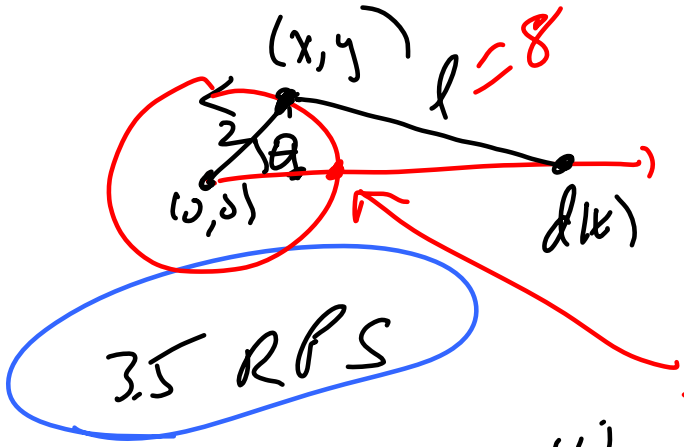
$y(E) = b\sin E$

Find slope at (x_0, y_0)

$$\text{slope} = \frac{\frac{dy}{dE}}{\frac{dx}{dE}} = \frac{b\cos E}{-a\sin E} = \frac{\frac{b}{a}x_0}{-a\frac{y_0}{b}}$$

$$= -\frac{b^2}{a^2}\frac{x_0}{y_0}$$

HW Prob:



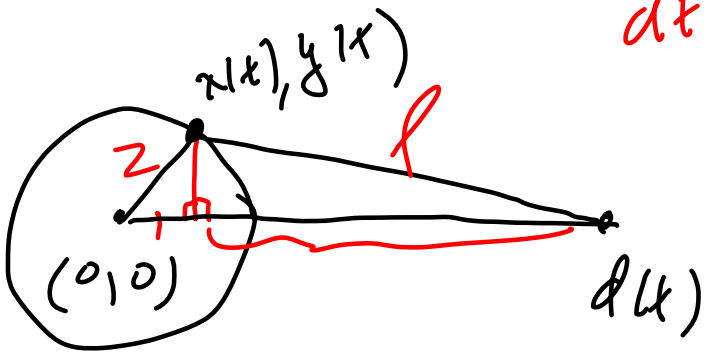
3.5 RPS

at $t=0$ Then $\theta_0 = 0$

$x(t) = 2\cos\theta(t)$

$y(t) = 2\sin\theta(t)$

Find $d'(t)$



$$\frac{d\theta}{dt} = \frac{3.5 \text{ rev}}{\text{sec.}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}}$$

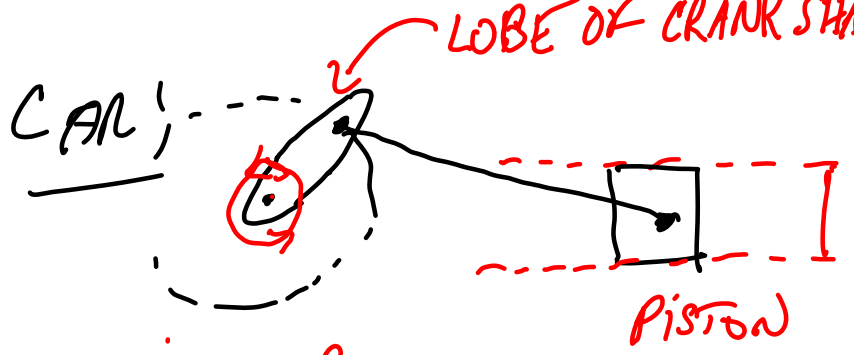
$$\theta = (3.5 \cdot 2\pi)t + \theta_0$$

$$= 7\pi t$$

$\theta_0 = 0$.

$$d(t) = 2\cos\theta(t) + \sqrt{l^2 - [2\sin\theta(t)]^2}$$

$$d(t) = 2\cos 7\pi t + \sqrt{64 - 4\sin^2(7\pi t)}$$



OR.

STEAM ENGINE