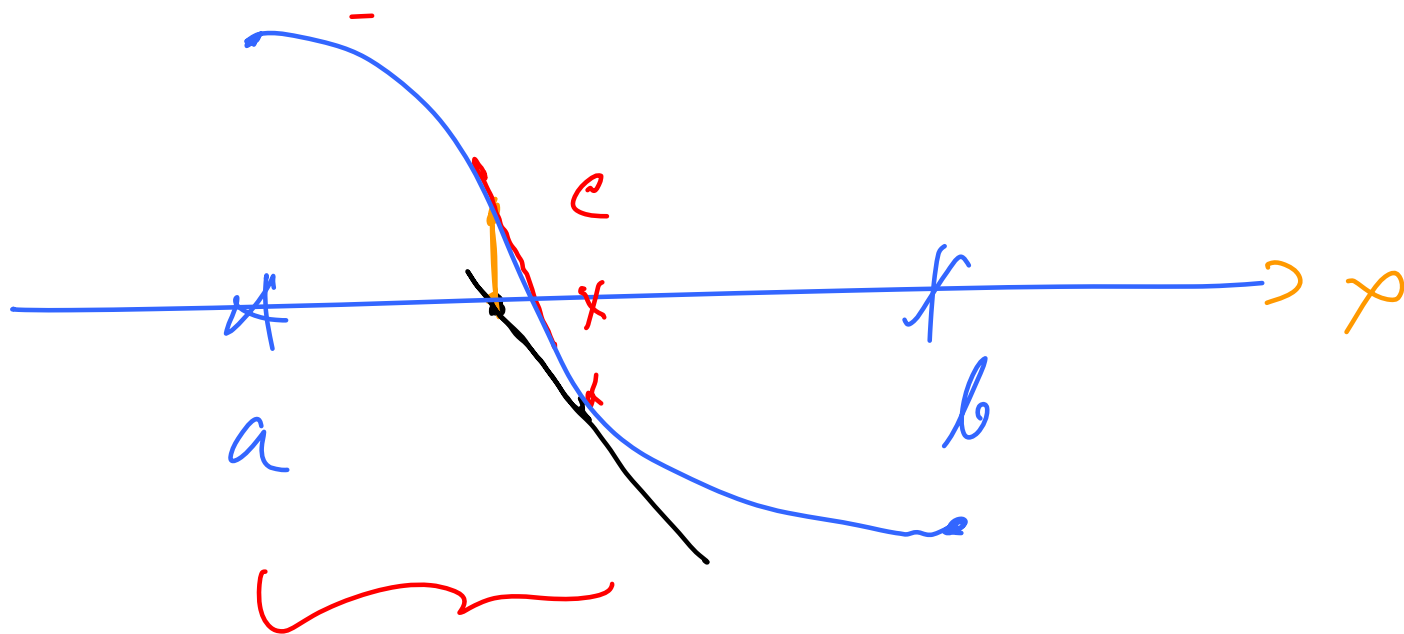
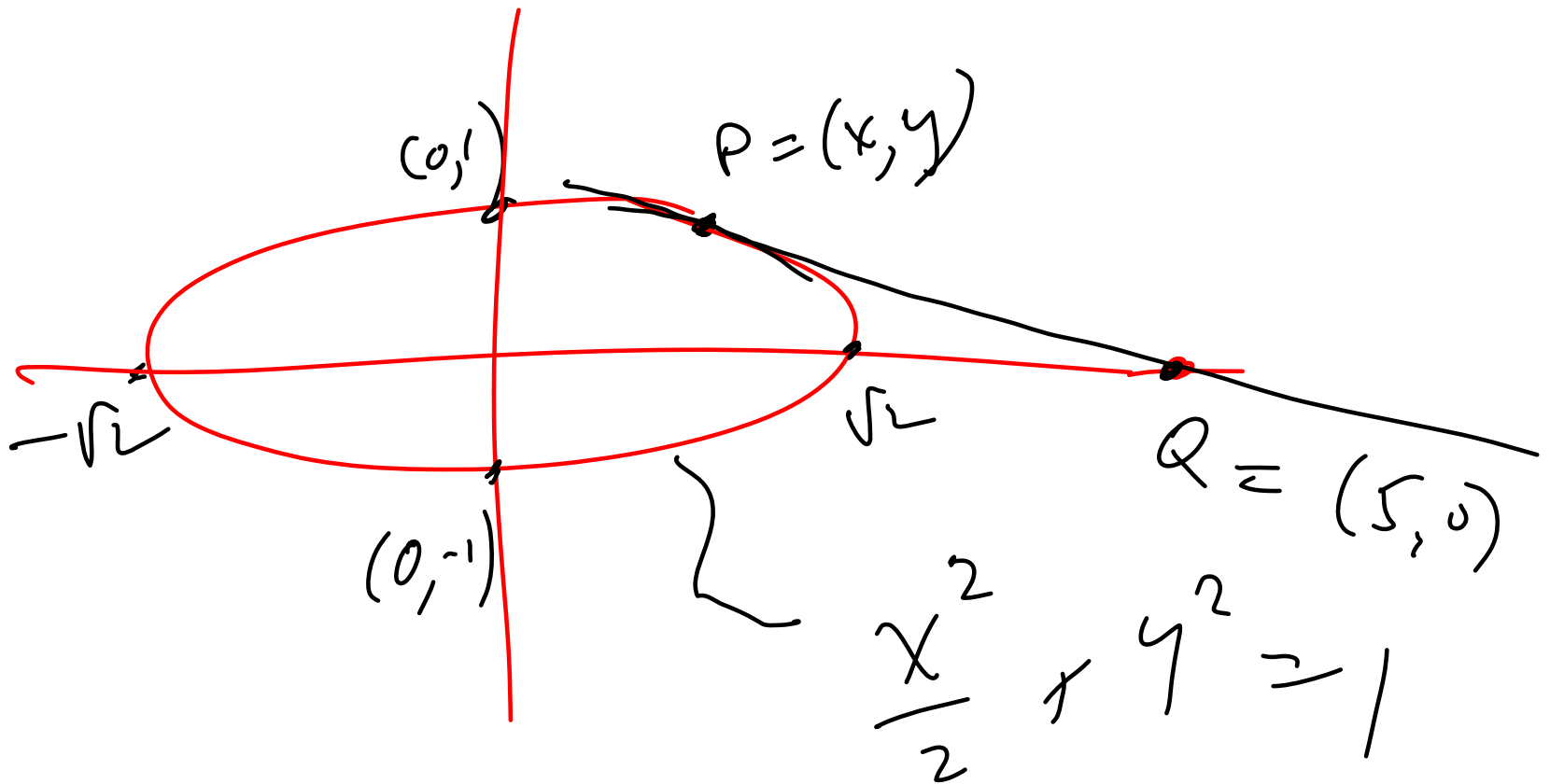


IVT

$$y = 3x - 4$$



$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x^2 \sin\left(\frac{1}{x}\right) + 2x} = \lim_{h \rightarrow 0} \frac{e^h - 1}{h \left[ h \sin\left(\frac{1}{h}\right) + 2 \right]} \approx \frac{1}{2}$$



$$y^2 = 1 - \frac{x^2}{2}$$

$$y = \sqrt{1 - \frac{x^2}{2}}$$

$$\frac{-\sqrt{1 - \frac{x^2}{2}}}{5 - x} = \frac{\frac{1}{2}(-x)}{\sqrt{1 - \frac{x^2}{2}}}$$

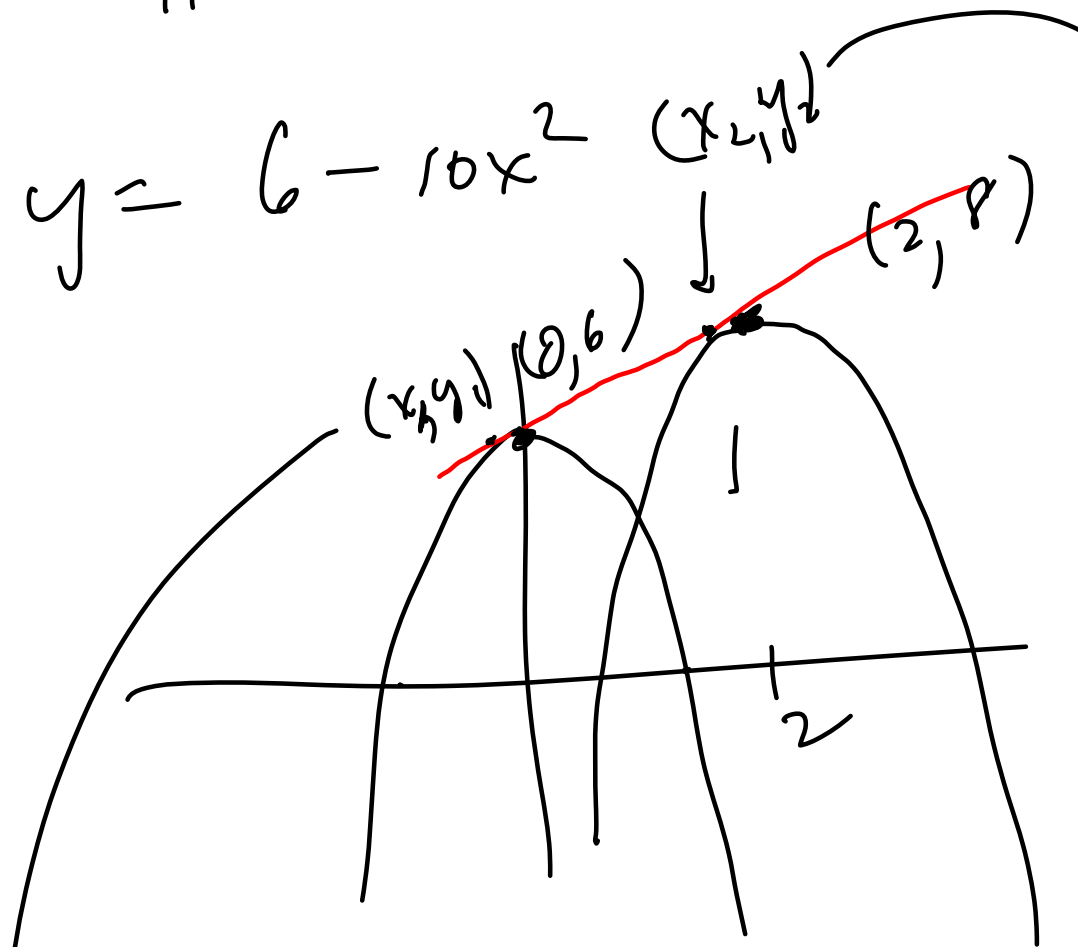
$\frac{d}{dx} e^x$  when  $x=0$  is

$$\lim_{h \rightarrow 0} \frac{e^h - e^0}{h} = 1$$

So  $\frac{e^x - 1}{x} \rightarrow 1$

$$y_1 = 6 - 10x_1^2$$

$$y_2 = 8 - (x_2 - 2)^2$$



$$y = 6 - 10x^2$$

$$y = 8 - (x - 2)^2$$

$(x_1, y_1)$   
 $(0, 6)$

$(x_2, y_2)$   
 $(2, 8)$

$$y' = -20x$$

slope =  $-20x_1$

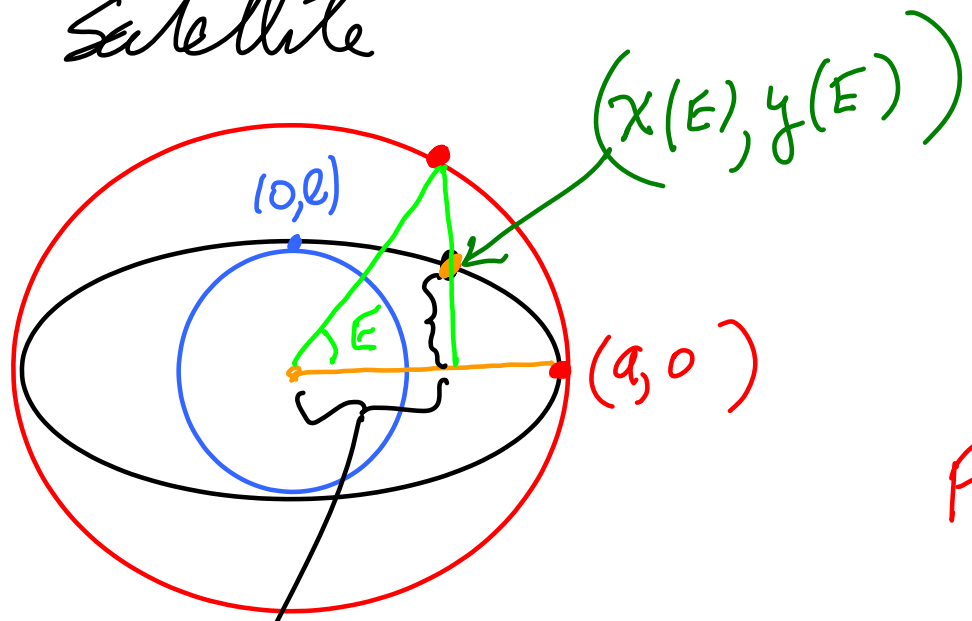
slope  $y' = -2(x - 2)$   
at  $x_2$  slopes

$$-2(x_2 - 2)$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

planet or satellite

10/24



$$\text{ellipse } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$E = \text{eccentric anomaly}$

(red, blue are circles)  
black is ellipse

Find  $x(E), y(E)$

$$a \cos(E) = x(E)$$

$$\left( \frac{a \cos E}{a} \right)^2 + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - \cos^2 E = \sin^2 E$$

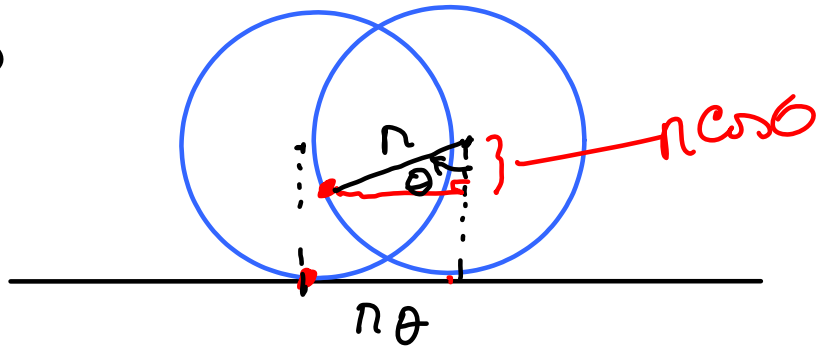
$$y = \pm b \sin E$$

$E$  is a "parameter"

the ellipse is a parametric curve

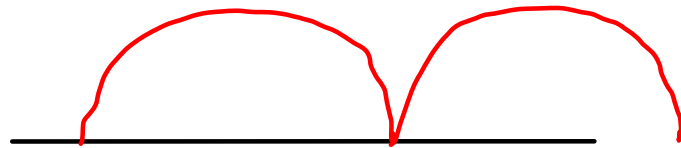
$$\text{So } y(E) = b \sin E$$

# CYLOID

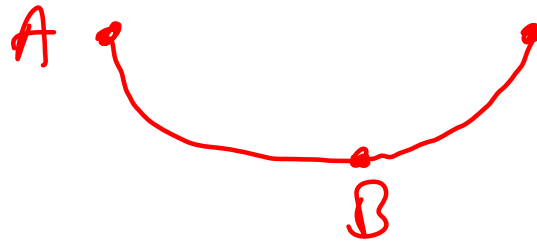


$$y = r - r \cos \theta$$
$$x = r\theta - r \sin \theta$$

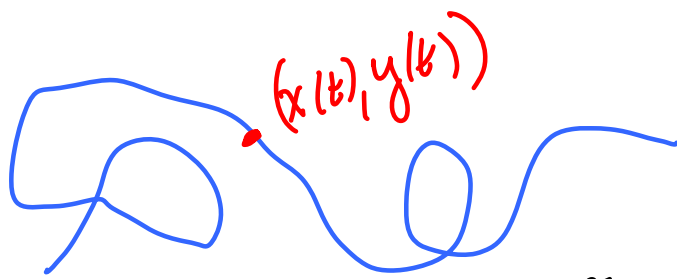
Trace red point



FLIP



FASTEST FROM A TO B



a "curve" not a graph.

$t = \text{time}$

$$x_1 = 4 \sin t \quad y_1 = 2 \cos t$$

$$x_2 = -4 + \cos t \quad y_2 = 1 + \sin t$$

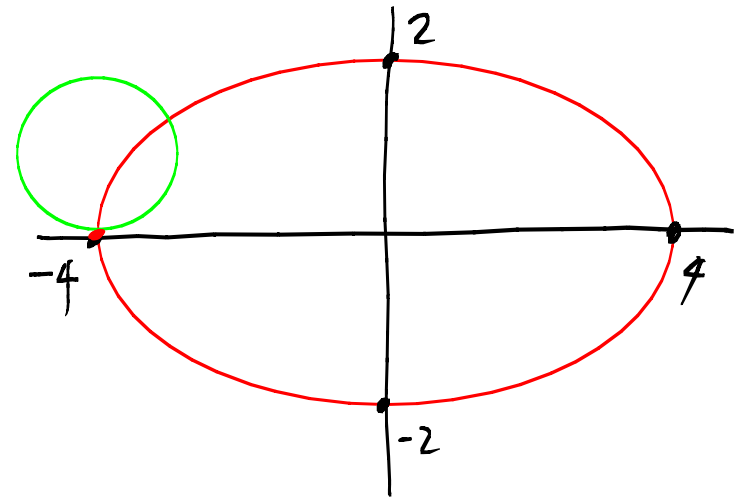
what are these curves?

$(x_1, y_1)$  parametrized curve

$(x_2, y_2)$  " "

$$\rightarrow \left(\frac{x_1}{4}\right)^2 + \left(\frac{y_1}{2}\right)^2 = 1$$

$$\rightarrow (x_2 + 4)^2 + (y_2 - 1)^2 = 1$$



Do they intersect?

If parametrized curves represent particles at time  $t$ , Do they collide?

Reword: Is there a time  $t$  with  $x_1 = x_2$  and  $y_1 = y_2$ ?

①  $4 \sin t = -4 + \cos t$

②  $2 \cos t = 1 + \sin t$

$2 \cos t - 1 = \sin t$

$4(2 \cos t - 1) = -4 + \cos t$

$8 \cos t - 4 = -4 + \cos t$

$8 \cos t = \cos t$

$7 \cos t = 0$

$\cos t = 0$

$4 \sin t = -4$

$\sin t = -1$

$t = \frac{3\pi}{2} + k2\pi$   
 $k$  integer