10-21-16

REVIEW

- LIMITS - RULES

VERTICAL, HORIZONTAL ASYMPTOTES.

$$\lim_{x\to a} \qquad \lim_{x\to a^+} \qquad \lim_{x\to a^-}$$

- SOMETIMES NEED TO CHANGE FORM TO COMPUTE LIMIT

e.g. $$\lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

- CHORDAL SLOPE $\qquad \dfrac{f(x+h)-f(x)}{h}$ or $\dfrac{f(b)-f(a)}{b-a}$

- SLOPE $\qquad \displaystyle\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ or $\displaystyle\lim_{b\to a} \frac{f(b)-f(a)}{b-a}$

AVERAGE SPEED ON $[a,b]$

(INSTANTANEOUS) SPEED.

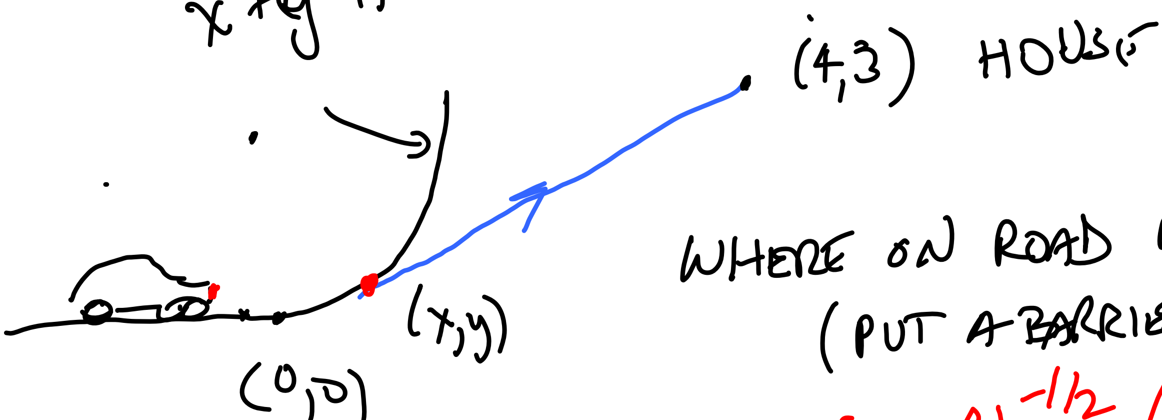
f' AS A FUNCTION

DRAW GRAPH OF f' FROM GRAPH OF f.

- DIFF. RULES $\quad + \; - \; \times \; \div \; \circ$
- DERIV. OF x^p, e^x, TRIG FUNCTIONS
- FIND EQ. OF TANGENT LINE

$x^2+(y-1)^2=1$

$$(y-1)^2 = 1-x^2$$
$$y-1 = \pm\sqrt{1-x^2}$$
$$y = 1 \pm \sqrt{1-x^2}$$
$$\boxed{y = 1-\sqrt{1-x^2}}$$



WHERE ON ROAD WILL LIGHTS HIT HOUSE?
(PUT A BARRIER NEAR THERE)

$$\text{slope} = -\frac{1}{2}\left(1-x^2\right)^{-1/2}(-2x) = \frac{x}{\sqrt{1-x^2}}$$

$$\text{Slope} = \boxed{\frac{x}{\sqrt{1-x^2}} = \frac{3-(1-\sqrt{1-x^2})}{4-x}}$$

$(4,3)$

(x, y)

$y = 1-\sqrt{1-x^2}$

$$4x - x^2 = (2+\sqrt{1-x^2})\sqrt{1-x^2}$$

$$4x - \cancel{x^2} = 2\sqrt{1-x^2} + 1 - \cancel{x^2}$$

$$4x - 1 = 2\sqrt{1-x^2}$$

$$16x^2 - 8x + 1 = 4(1-x^2)$$

$$20x^2 - 8x - 3 = 0$$

$$x = \frac{8 \pm \sqrt{64+240}}{2 \cdot 20}$$

$$= \frac{8 \pm \sqrt{304}}{40}$$

$$= \frac{8 \pm 4\sqrt{19}}{40}$$

$$= \frac{2 \pm \sqrt{19}}{10}$$

$4\sqrt{304}$, $4\sqrt{76}$, 19

$$x = \frac{2+\sqrt{19}}{10} \quad y = 1 - \sqrt{1-x^2}$$

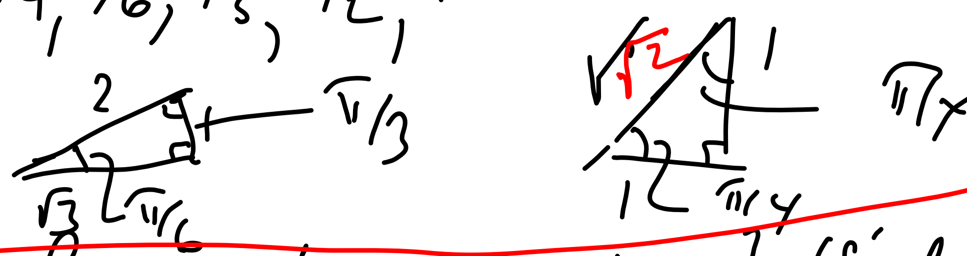
ESTIMATE $\tan\left(\frac{\pi}{4} + .001\right)$ without using a calculator.

(i.e. find it approximately)

$f(x) = \tan x$ at $x = \pi/4$ $f(\pi/4) = 1$

$f'(x) = \sec^2 x$ $f'(\pi/4) = 2$

$0, \pi/4, \pi/6, \pi/3, \pi/2, \pi$

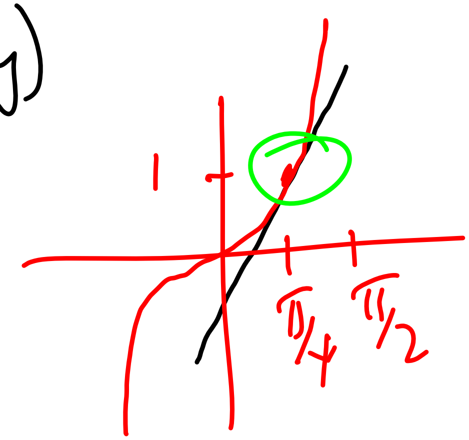


$y = 2(x - \pi/4) + 1$

$y\left(\frac{\pi}{4} + .001\right) = .002 + 1$

$= 1.002$

Where is $\tan x = 1.01$? (find x approximately)



When is $2(x - \pi/4) + 1 \overset{?}{=} 1.01$

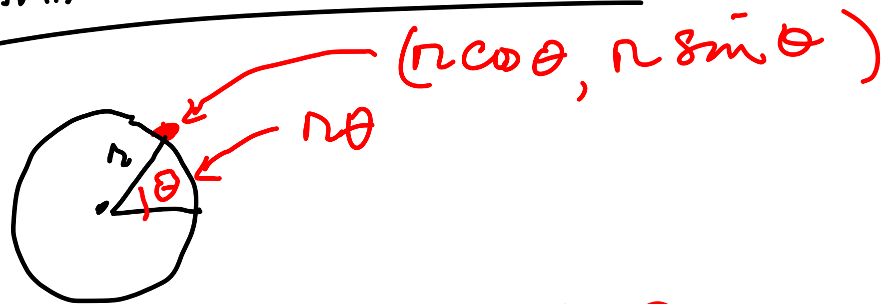
$2(x - \pi/4) = .01$

$x - \pi/4 = .005$

$x = \frac{\pi}{4} + .005$

10/22/14

PARAMETRIC EQUATIONS



$x = r\cos\theta$

$y = r\sin\theta$

Suppose θ is a function of time

$x = r\cos\theta(t)$ now x & y

$y = r\sin\theta(t)$ are functions of t

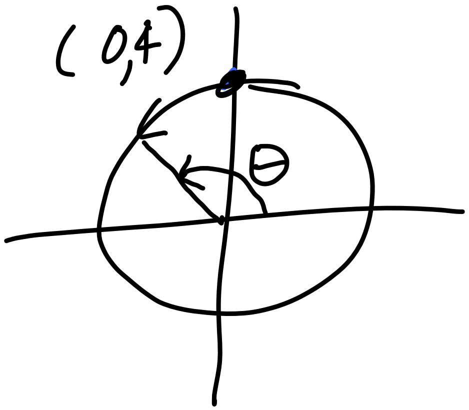
$\frac{d\theta}{dt}$ is called the ANGULAR VELOCITY

= rate of change of θ with respect to time

θ measured in radians so if t measured in seconds

$d\theta/dt$ radians per second

SUPPOSE ANGULAR VELOCITY IS 5 rad per sec ; radius = 4
START AT (0,4). WHAT IS y COORDINATE t seconds later?

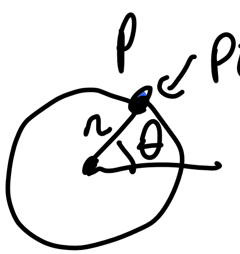


$$y = 4\sin\theta(t)$$

$$\theta(t) = \frac{\pi}{2} + 5t$$

$$y(t) = 4\sin\left(\frac{\pi}{2} + 5t\right)$$

CAR ENGINE ROTATING AT 3600 RPM (rev. per min)



POINT ON CRANKSHAFT. Find $P(t)$. t = time in seconds.

$$\theta(t) = \left[3600\,\frac{2\pi\,\text{rad}}{\cancel{\text{min}}}\right] t \times \frac{1\,\cancel{\text{min}}}{60\,\text{sec}}.$$

$$= 120\pi\,\frac{\text{rad}}{\text{sec}} \cdot t\,\text{sec}$$

$$x = r\cos(120\pi t) \qquad y = r\sin(120\pi t)$$