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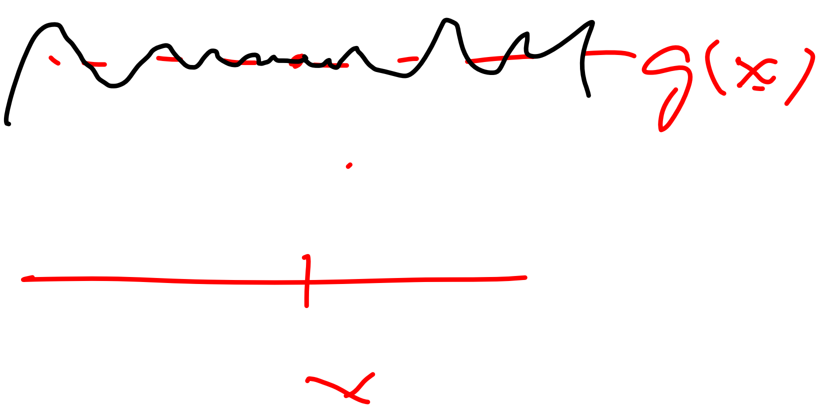
$g(x+h) - g(x) = k$

$k \to 0$ as $h \to 0$ BECAUSE g is CONTINUOUS

$g(x+h) = g(x) + k$

$$= \underbrace{\frac{f(g(x)+k) - f(g(x))}{k}}_{\downarrow \atop f'(g(x))} \cdot \underbrace{\frac{g(x+h)-g(x)}{h}}_{\downarrow \atop g'(x)}$$



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$$\downarrow \qquad\qquad \downarrow$$

$$f'(g(x)) \qquad\qquad g'(x)$$

works if $g(x+h) \neq g(x)$

i.e. TRUE IF WE ONLY ALLOW THOSE h WITH $g(x+h) \neq g(x)$

If $g(x+h) - g(x) = 0$ HAS SOLUTIONS WITH ARBITRARILY SMALL h

THEN $g'(x) = 0$. AND LEFT SIDE $= 0$ SO STILL TRUE.

MORE EXAMPLES:

FROM W2014 FINAL EXAM:

DIFFERENTIATE

$$y = \sqrt{\sin(\pi x) + 3x^2}$$

$$y' = \frac{1}{2}\left(\sin(\pi x) + 3x^2\right)^{-1/2}\left[\cos(\pi x)\cdot\pi + 6x\right]$$

a, b CONSTANTS

$$y = e^{-ax^2}\cos\left(\frac{\pi}{b}x\right)$$

$$y' = \left[e^{-ax^2}\right]\left\{-2ax\right\}\cos\left(\frac{\pi}{b}x\right) + e^{-ax^2}\left[-\sin\left(\frac{\pi}{b}x\right)\frac{\pi}{b}\right]$$

SP 2014 FINAL

$$y = x^3 3^x$$

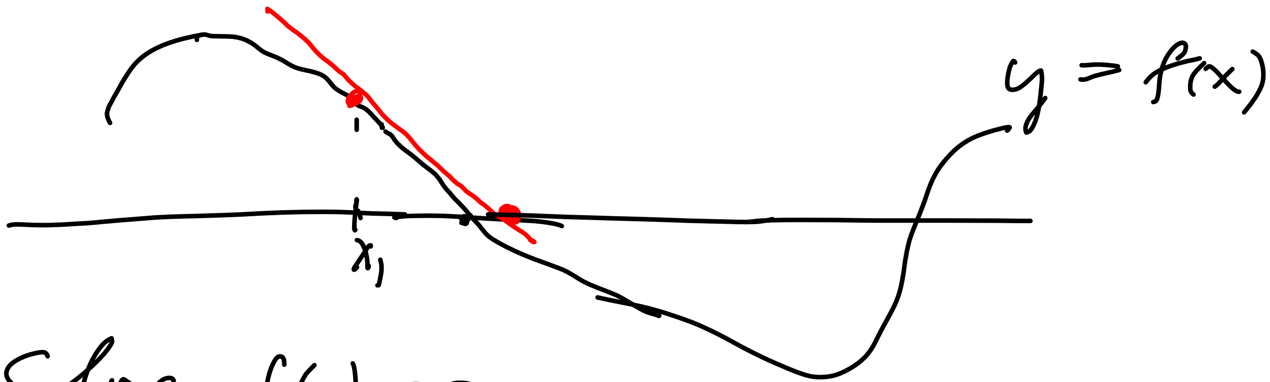
$$3 = e^{\ln 3} \qquad 3^x = e^{(\ln 3)x}$$

$$y' = 3x^2 3^x + x^3\left[3^x \ln 3\right]$$

$$y = \sqrt{x + \cos(x + e^x)}$$

$$y' = \frac{1}{2}\left(x + \cos(x + e^x)\right)^{-1/2}\left[1 + \left(-\sin(x+e^x)\right)\cdot(1+e^x)\right]$$

APPLICATION



Want to solve $f(x) = 0$

Find tangent line when $x = x_1$ & see where it crosses x axis instead.

example

Solve $x^5 - 4x = 2$

$f(x) = x^5 - 4x - 2$

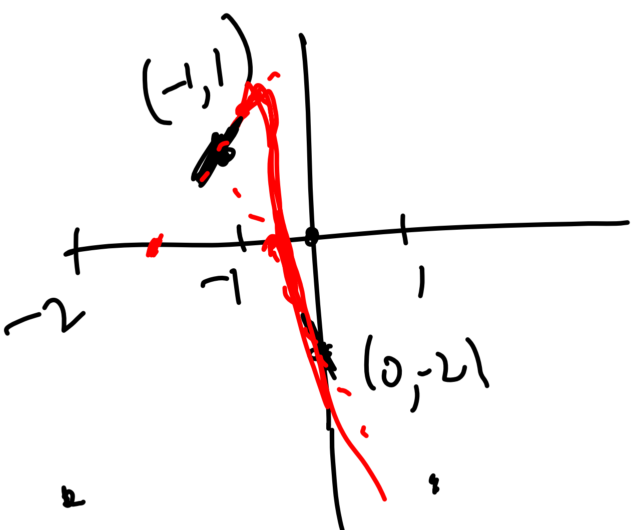
$f' = 5x^4 - 4$

$f(-1) = 4 - 3 = 1$

$f(0) = -2$

$f'(-1) = 5 - 4 = 1$

$f'(0) = -4$



Tang line when $x=0$ $\quad f(x)=x^5-4x-2$

$\boxed{y=-4x-2}$

where is $y=0$? $\quad x=-1/2$

$f(-1/2)=-\frac{1}{32}+2-2=-1/32=-.03125$

REPEAT:

Tangent line when $x=-1/2$

$f'(x)=5x^4-4=\frac{5}{16}-4=\frac{-59}{16}$

$y=\frac{-59}{16}\left(x+\frac{1}{2}\right)-1/32$

Where is $y=0$? $\quad x+1/2=\frac{-16}{59}\cdot\frac{1}{32}=-\frac{1}{108}$

$x=-1/2-\frac{1}{108}$

$f\left(-\frac{1}{2}-\frac{1}{108}\right)=-\left(\frac{1}{2}+\frac{1}{108}\right)^5-4\left(\frac{1}{2}-\frac{1}{108}\right)-2$

$=-.0342526+.037037=.002\ldots$

- SAMPLE MIDTERM AVAILABLE THURS. AFTERNOON (SEE WEB PAGE).
- <u>READ</u> <u>COVER SHEET</u> <u>CAREFULLY</u>
- WRONG CALCULATOR IS GROUNDS FOR FAILURE
- SOLUTIONS TO SAMPLE M.T. WILL <u>NOT</u> BE PROVIDED

  A GOAL FOR THIS CLASS IS FOR YOU TO DEVELOP THE ABILITY TO DECIDE FOR YOURSELF WHETHER A SOLUTION IS CORRECT OR NOT.

- THE MIDTERM PROBLEMS MAY NOT BE SIMILAR
  THIS IS JUST A SAMPLE OF THE TYPES OF PROBLEMS

- PARTIAL CREDIT WILL BE GIVEN
- GRADING IS NOT LIKE QUIZZES & HW
  WE ARE LOOKING AT THE LOGIC FOR EACH STEP