10/17/16 QUESTIONS?

let $g(x) = f(ax+b)$

Then $g'(x) = f'(ax+b) \cdot a$

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Then $g'(x) = f'(ax+b) \cdot a$

PROOF:

$$\frac{g(x+h) - g(x)}{h} = \frac{f(\overbrace{ax+ah+b}^{a(x+h)+b}) - f(\overbrace{ax+b}^{x_1})}{h}$$

let $x_1 = ax+b$

$h_1 = ah$

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Then $g'(x) = f'(ax+b) \cdot a$

<u>PROOF</u>:

$$\frac{g(x+h)-g(x)}{h} = \frac{f(ax+ah+b)-f(ax+b)}{h}$$

let $x_1 = ax+b$

$h_1 = ah$

$$= \frac{f(x_1+h_1)-f(x_1)}{h_1/a} \xrightarrow[\text{as } h\to 0]{} \frac{f'(x_1)}{1/a} = a f'(x_1)$$

$$= a f'(ax+b)$$

//

$$\boxed{\frac{d}{dx}\left[f(ax+b)\right] = f'(ax+b)\cdot a}$$

$$\frac{d}{dx}(3x-2)^{80} = 3\cdot 80(3x-2)^{79}$$

$a=3 \quad b=-2$

$f(x) = x^{80} \qquad f'(x) = 80x^{79} \qquad f'(3x-2) = 80(3x-2)^{79}$

$$\frac{d}{dx}e^{3x-2} = e^{(3x-2)}3$$

$$\frac{d}{dx}10^x =$$

$10 = e^{\ln(10)}$

$10^x = e^{\ln(10)\cdot x}$

$\frac{d}{dx}10^x = e^{(\ln 10)x}\ln 10$

$= 10^x \ln 10$

$$\frac{d}{dx} a^x = \frac{d}{dx}\left[e^{\ln a}\right]^x = e^{(\ln a)x} \cdot \ln a = a^x \ln a$$

$$\frac{d}{dx} \sin(3x - \pi/4) = \cos\left(3x - \frac{\pi}{4}\right) \cdot 3$$

$$\frac{d}{dx} \frac{1}{\sin(3x-2)} = \frac{0 - 1 \cdot [\cos(3x-2) \cdot 3]}{\sin^2(3x-2)}$$

$$\frac{d}{dx}\left[e^{2x}\sin 3x\right] = e^{2x}\left[\{\cos(3x)\}3\right] + 2e^{2x}\sin 3x$$

$$\frac{d}{dx}\left(x^2 e^x + 4\sin(5x+1)\right)$$

$$= \left[2x e^x + x^2 e^x\right] + 4\left[\cos(5x+1)\right]\cdot 5$$

Composition f, g new function: $f \circ g$

$$(f \circ g)(x) = f(g(x))$$

- EXAMPLE $f(x) = e^x$ $g(x) = \sin x$

$$(f \circ g)(x) = e^{\sin x}$$

$$(g \circ f)(x) = g(f(x)) = \sin(e^x)$$

- IF $h(x) = \tan(3x-2)$ find f & g so that $\underline{h = f \circ g}$

$f(x) = \tan x$
$g(x) = 3x - 2$

- $3(\cos(5x-2))^6$ "outside function" $= 3x^6$
"inside function" $= \cos(5x-2)$

$$\frac{1}{[\tan x]^2}$$

$g(x) = \tan x$

$f(x) = x^2$

$h(x) = \frac{1}{x}$

$h[f(g(x))]$

or $k(x) = x^{-2}$

$k(g(x))$

CHAIN RULE:

If $h(x) = f(g(x))$

then $h'(x) = f'(g(x))\,g'(x)$ (chain rule)

examples: • $h(x) = \sqrt{\tan x}$

$$h'(x) = \frac{1}{2}(\tan x)^{-1/2} \sec^2 x$$

• $h(x) = e^{3x-2}$

$$h'(x) = e^{3x-2} \cdot 3$$

• $h(x) = 3(\cos(5x-2))^6$

$$h'(x) = 18(\cos(5x-2))^5 [-\sin(5x-2)]5$$

KEY TO CHAIN RULE: RECOGNIZE HOW THE FUNCTION IS BUILT FROM SIMPLER FUNCTIONS.

$$h(x) = \sqrt{x + \sqrt{x^2+1}}$$

$$= \left(x + (x^2+1)^{\frac{1}{2}}\right)^{1/2}$$

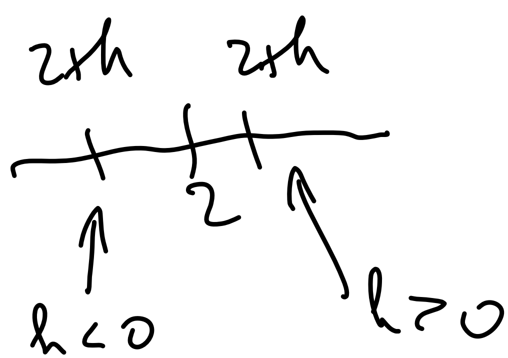
$$h'(x) = \frac{1}{2}\left(x+(x^2+1)^{\frac{1}{2}}\right)^{-1/2}\left[1 + \frac{1}{2}(x^2+1)^{-1/2}2x\right]$$

$$f(x) = \begin{cases} ax & 0 \le x \le 2 \\ e^{cx} & 2 < x < \infty \end{cases}$$

Find a and c so that f is differentiable at $x = 2$.

$$f'(x) = \begin{cases} a & 0 \le x < 2 \\ c e^{cx} & 2 < x < \infty \end{cases}$$

f cont $\Rightarrow$ $2a = e^{2c}$ so $a = \frac{1}{2} e^{2c}$



need $a = c e^{2c}$

So $c = 1/2$

$a = \frac{1}{2} e$

FIND ALL x WHERE THE GRAPH OF $y = e^{2x} \sin 3x$

HAS A HORIZONTAL TANGENT.

$$y' = 2e^{2x} \sin 3x + e^{2x} 3 \cos(3x)$$

$$y' = 0 \qquad 2 \sin 3x + 3 \cos 3x = 0$$

$$\frac{2}{3} \tan 3x + 1 = 0$$

$$\tan 3x = -\frac{3}{2}$$

$$3x = \tan^{-1}(-3/2) + k\pi$$

$$x = \frac{1}{3} \tan^{-1}(-3/2) + \frac{k\pi}{3}$$