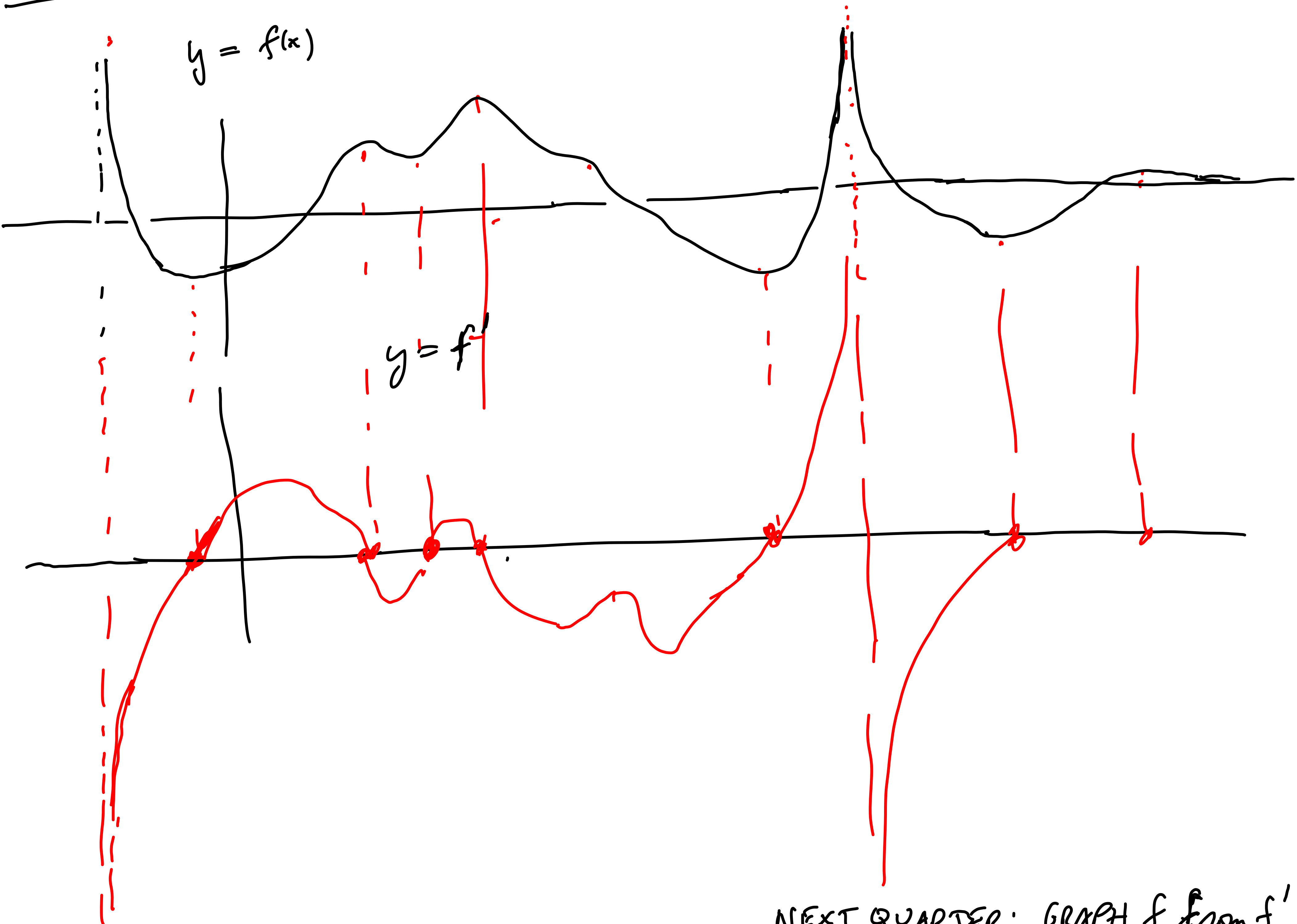
10-12-16

Hint on these problems: Draw picture first then look for answer



NEXT QUARTER: GRAPH f from f'

PLOT: FIND THE DERIVATIVES OF A FEW FUNCTIONS, THEN USE $+ \; - \; \times \; \div$ AND COMPOSITION TO MAKE MORE COMPLICATED FUNCTIONS

If $f(x) = 1$ then $f'(x) = 0$

If $f(x) = x$ then $f'(x) = 1$

Recall: $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

PLOT: FIND THE DERIVATIVES OF A FEW FUNCTIONS, THEN USE $+ - \times \div$ AND COMPOSITION TO MAKE MORE COMPLICATED FUNCTIONS

If $f(x) = 1$ then $f'(x) = 0$

If $f(x) = x$ then $f'(x) = 1$

Rules:

1. $(f \pm g)' = f' \pm g'$

2. If c is CONSTANT

$$(cf)' = cf'$$

3. $(fg)' = f'g + fg'$ (NOT $f'g'$)

4. $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$ (not f'/g')

WHY?

PROOF OF 1: $(f+g)' = f' + g'$

$$\frac{f(x+h) + g(x+h) - [f(x) + g(x)]}{h}$$

$$= \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$= \underbrace{\frac{f(x+h) - f(x)}{h}} + \frac{g(x+h) - g(x)}{h} \xrightarrow[\text{as } h \to 0]{} f'(x) + g'(x)$$

PROOF OF 2: $(cf)' = cf'$

$$\frac{cf(x+h) - cf(x)}{h} = c\left[\frac{f(x+h) - f(x)}{h}\right] \longrightarrow cf'(x)$$

PROOF OF 3; $(fg)' = f'g + fg'$

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \frac{[f(x+h) - f(x)]}{h} g(x+h) + f(x)\frac{[g(x+h) - g(x)]}{h}$$

$$\xrightarrow[\text{as } h \to 0]{} f'(x)g(x) + f(x)g'(x)$$

PROOF OF 4:

CLAIM: $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

PROOF:

$$\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \frac{\frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)}}{h}$$

$$= \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h\, g(x+h)g(x)}$$

$$= \frac{[f(x+h) - f(x)]g(x) - f(x)[g(x+h) - g(x)]}{h\, g(x+h)g(x)}$$

$$= \frac{\frac{[f(x+h) - f(x)]}{h}g(x) - f(x)\left[\frac{g(x+h) - g(x)}{h}\right]}{g(x+h)g(x)}$$

$$\longrightarrow \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx} x^n = (x^n)' = n x^{n-1} \qquad n = 1, 2, 3, 4, \ldots$$

<u>Proof:</u> Suppose we know it is true for $n = k$.

Then $\frac{d}{dx} x^{k+1} = \frac{d}{dx} (x^k \cdot x)$

$= k x^{k-1} \cdot x + x^k \cdot 1$ product rule

$= k x^k + x^k$

$= (k+1) x^k \leftarrow (n = k+1)$

<u>So:</u> If it is true when $n = k$ then true for $n = k+1$

Start: true for $n = 1$ $\qquad \frac{d}{dx} x = 1$

so true for $n = 2$, then true for $n = 3$, etc. (Induction)

$$\frac{d}{dx} x^{-k} = \frac{d}{dx} \frac{1}{x^k} = \frac{x^k \cdot 0 - 1 \cdot (k x^{k-1})}{x^{2k}}$$

$$= \frac{-k x^{k-1}}{x^{2k}} = -k x^{-k-1}$$

so $\frac{d}{dx} x^n = n x^{n-1}$ for n any integer, pos or neg.
Section 3.6 we'll prove it is true for any real number n

$$\frac{d}{dx} x^{37} = 37 x^{36}$$

$$\frac{d}{dx} x^{-3} = -3 x^{-4}$$

$$\frac{d}{dx}\left[8x^4-7x^3+4x-5\right]=8\cdot 4x^3-7\cdot 3x^2+4\cdot 1-5\cdot 0$$

$$=32x^3-21x^2+4$$

$$\frac{d}{dx}\left(3x^2-8x\right)\left[7x^5-81\right)=\left(6x-8\right)\left(7x^5-81\right)$$

$$+\left(3x^2-8x\right)\left(35x^4\right)$$

$$\frac{d}{dx}\,\frac{3x^2-8x}{7x^5-81}=\frac{\left(7x^5-81\right)\left[6x-8\right]-\left[3x^2-8x\right]\left[35x^4\right]}{\left(7x^5-81\right)^2}$$

Def:

- $$e = \lim_{n\to\infty}\left(1+\frac{1}{2^n}\right)^{2^n} \approx 2.7182818$$

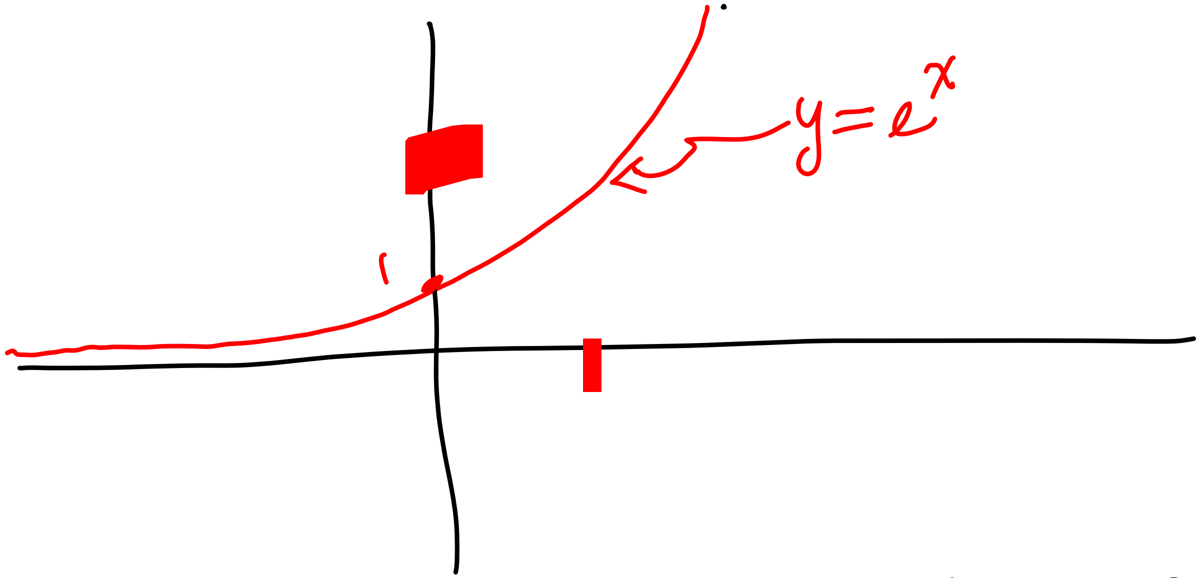
SEE COURSE WEB PAGE FOR A PROOF THAT LIM EXIST.
(USES SQUEEZE THEOREM)

- $$e^x e^y = e^{x+y}$$

- $$\lim_{h\to 0}\frac{e^h - 1}{h} = 1$$

- $$\lim_{h\to 0}\frac{e^{x+h}-e^x}{h} = \lim_{h\to 0}\frac{e^x e^h - e^x}{h}$$
  $$= \lim_{h\to 0} e^x\left[\frac{e^h-1}{h}\right] = e^x$$

- So $(e^x)' = e^x$



EACH HORIZONTAL LINE $y=c$, WITH $c>0$, HITS THE GRAPH ONCE. SO e^x HAS AN INVERSE FUNCTION: $\ln(x)$

$$\ln(e^x) = x$$

$$e^{\ln(x)} = x \qquad (x>0)$$

$$2 = e^{\ln 2} \qquad 2^x = e^{(\ln 2)x}$$

$$a^x = \left[e^{\ln a}\right]^x = e^{(\ln a)x}$$