10-10-16 2.7-2.8

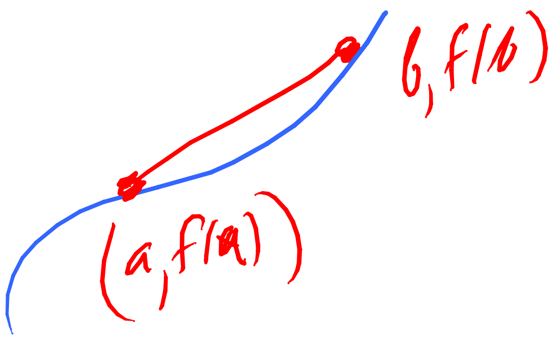
"RATE" = SLOPE

$y = f(x)$

OTHER NOTATION:

$$\underbrace{f'(a),\ y'(a)}_{\text{WE'LL USE THESE TWO}},\ \frac{df}{dx}(a),\ Df(a),\ \left.\frac{dy}{dx}\right|_{x=a}$$

KEY:



$$\lim_{b \to a} \frac{f(b) - f(a)}{b - a} = f'(a)$$

DIFF QUOT = "AVERAGE" RATE OF CHANGE

ITS LIMIT = "INSTANTANEOUS" RATE OF CHANGE

$$f(x) = \begin{cases} x^3 & 0 \le x \le 2 \\ c(x-2)+d & 2 < x < \infty \end{cases}$$

WHEN is f CONTINUOUS AT $x=2$?

$d=8$

WHEN IS THE <u>SLOPE</u> CONTINUOUS?

Slope of $y=x^3$ when $x=2$:

$$\frac{(x+h)^3 - x^3}{h} = \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h} = 3x^2 + 3xh + h^2$$

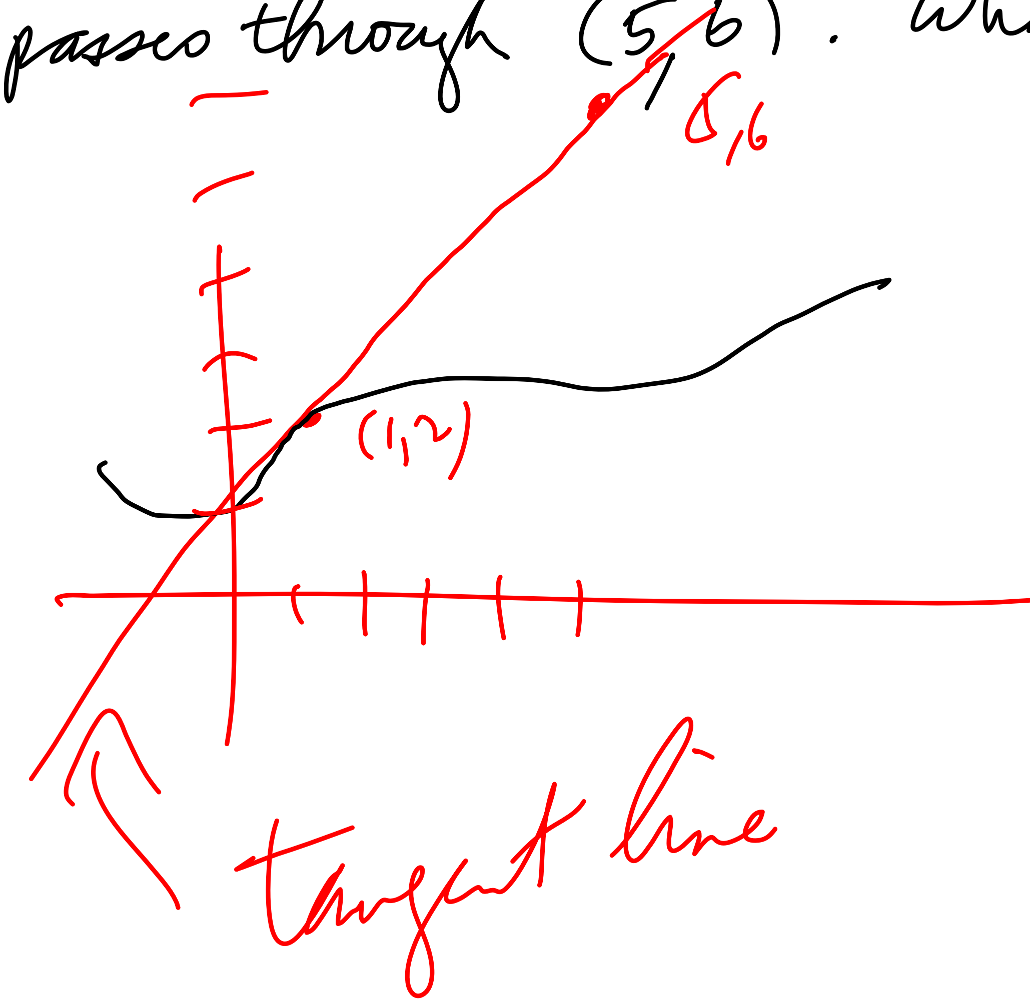
$$\longrightarrow 3x^2 \text{ as } h \to 0$$

when $x=2$ slope is $= 12$

$\underline{c = 12}$

Suppose the tangent line to $y = f(x)$ at $(1,2)$ passes through $(5,6)$. What is $f'(1)$?

1.



Suppose the tangent line to $y = f(x)$ at $(1,2)$ passes through $(5,6)$. What is $f'(1)$?

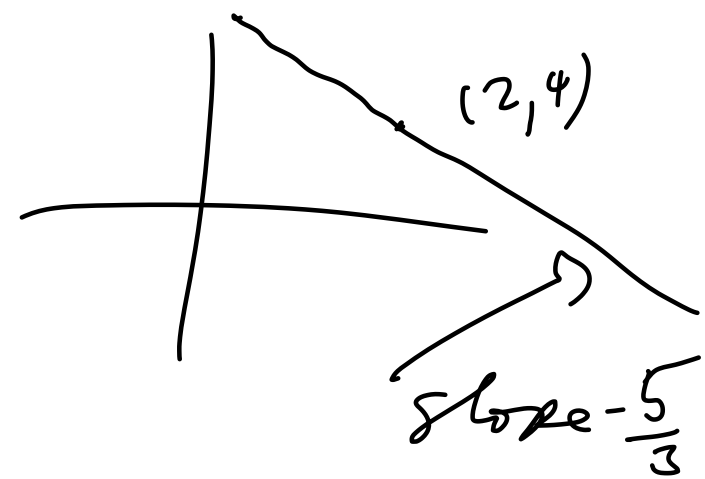
$P(t) = (x(t), y(t))$ where $x(t) = 2+3t$ $\quad y(t) = 4-5t$

$P(0) = (2,4)$

$P(1) = (5,-1)$

$t = \frac{x-2}{3}$ $\quad y = 4-5\left(\frac{x-2}{3}\right)$

Find y as a function of x.



t = time

$x'(t)$ = horizontal velocity =

$y'(t)$ = vertical velocity =

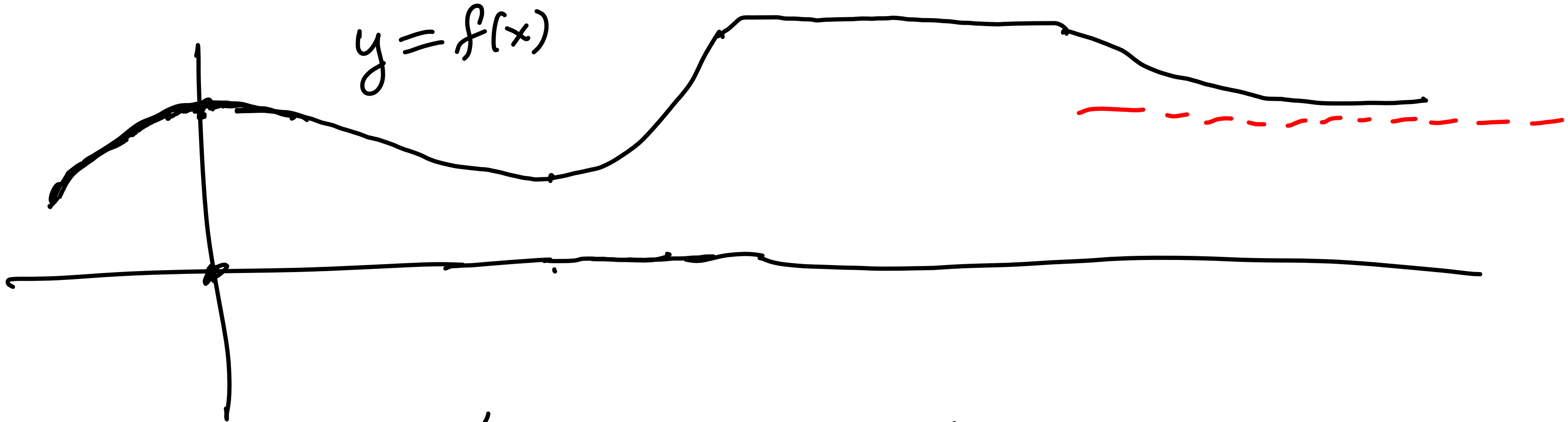
$f(x) = x^2$

$f'(x)$ is a new function - find it

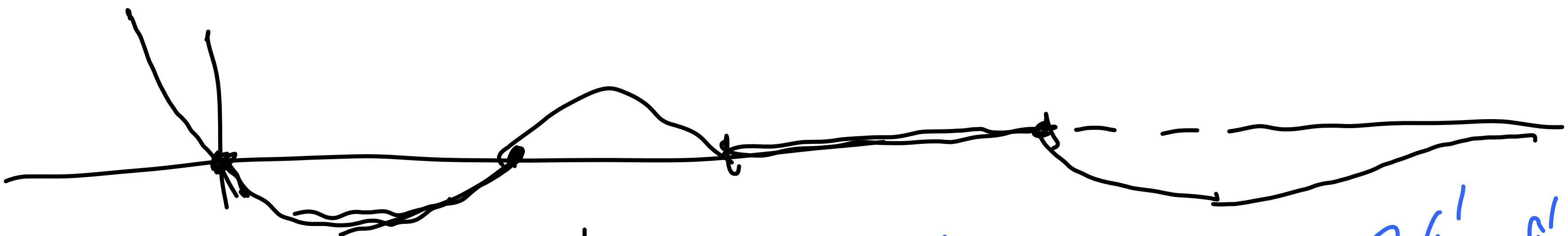
$$f'(a) = \lim_{x \to a} \frac{x^2 - a^2}{x - a} = \lim_{x \to a} x + a = 2a$$

$\therefore f'(a) = 2a$ for all a

i.e. $f'(x) = 2x$



GRAPH $y = f'(x)$ (ROUGHLY)



shift graph of f – how does it change f'? Between ZEROS of f', f' is > 0 or < 0 not both.

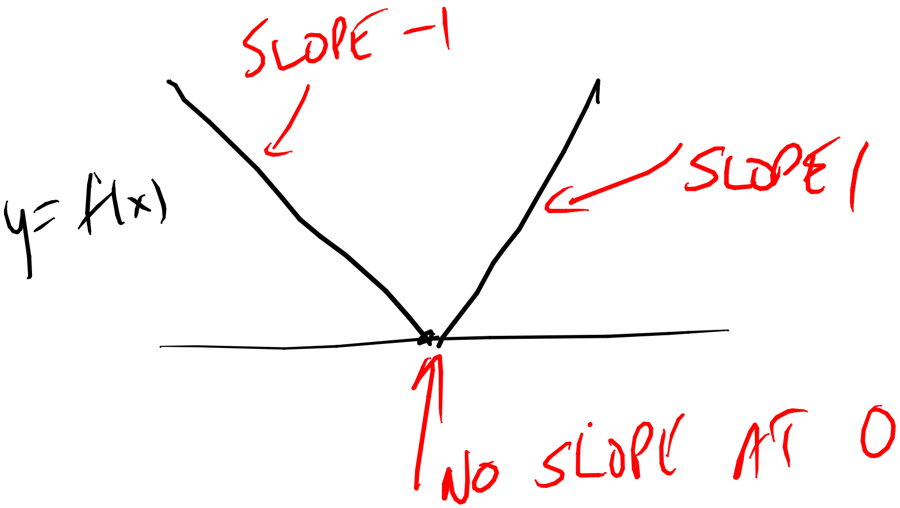
I.V.T.
because f' is contin.

Def: 1. f is <u>DIFFERENTIABLE</u> AT c if $f'(c)$ exists

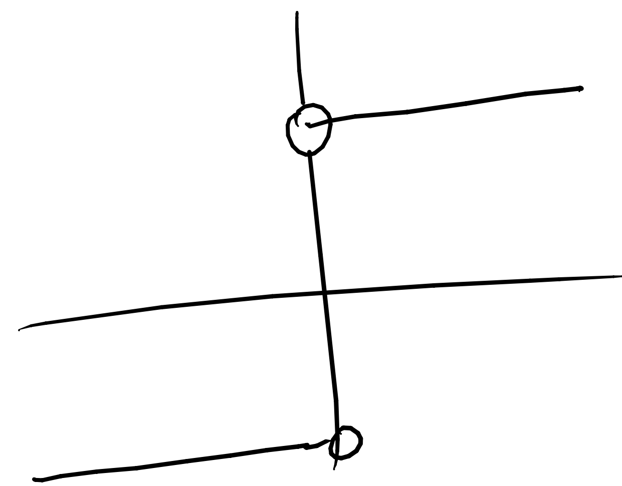
2. f is <u>DIFFERENTIABLE ON (a,b)</u> if f is DIFFERENTIABLE AT EACH c BETWEEN a & b

EXAMPLE FROM EARLIER LECTURE

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

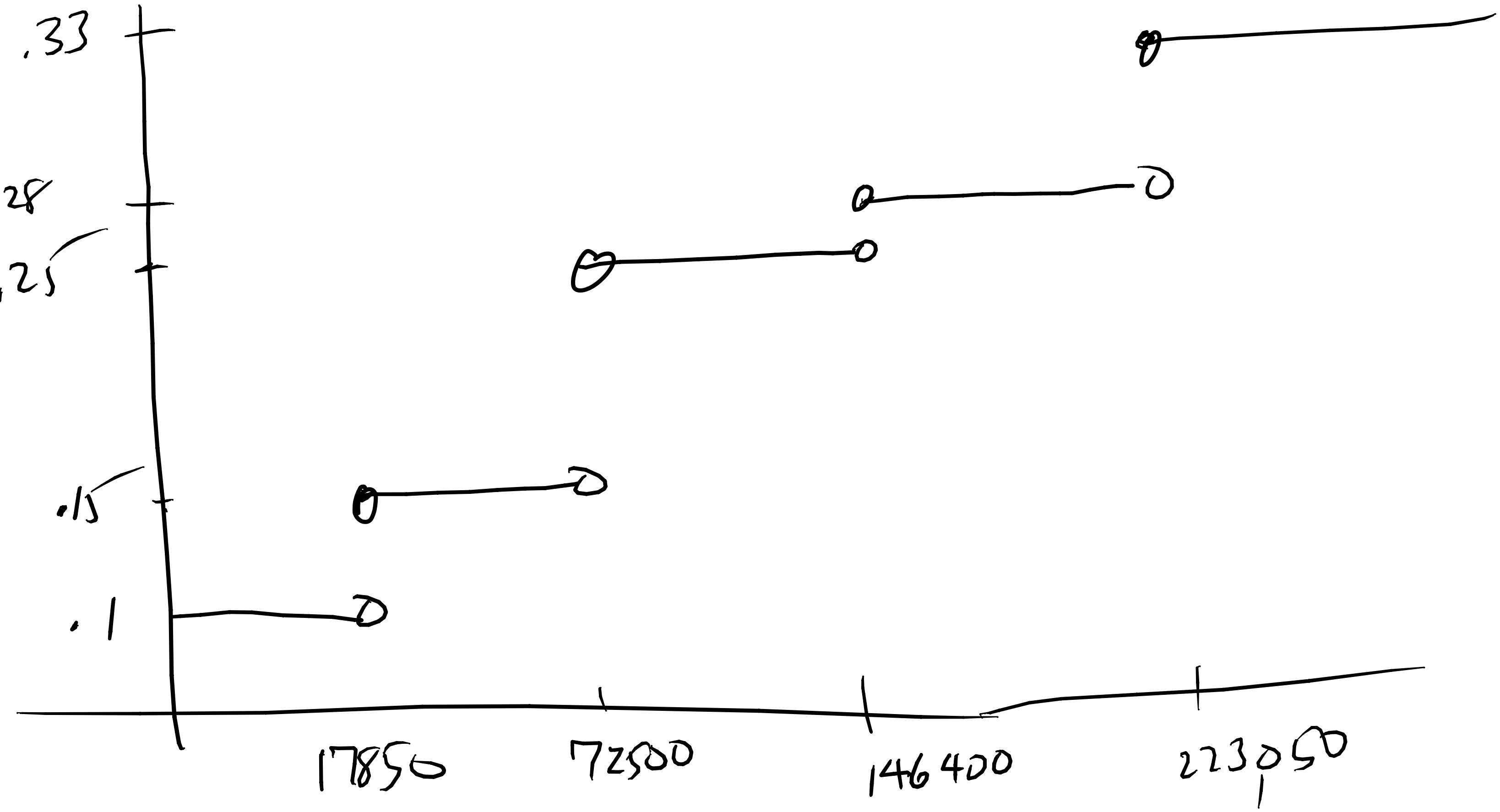


$f'(x)$:



f is CONTINUOUS ON $(-\infty,\infty)$; DIFFERENTIABLE ON $(-\infty,0)$ & $(0,\infty)$

GRAPH OF TAX RATES (NOT THE TAX)



#7 THEOREM

If $f'(a)$ exists then f is continuous at a

PROOF:

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

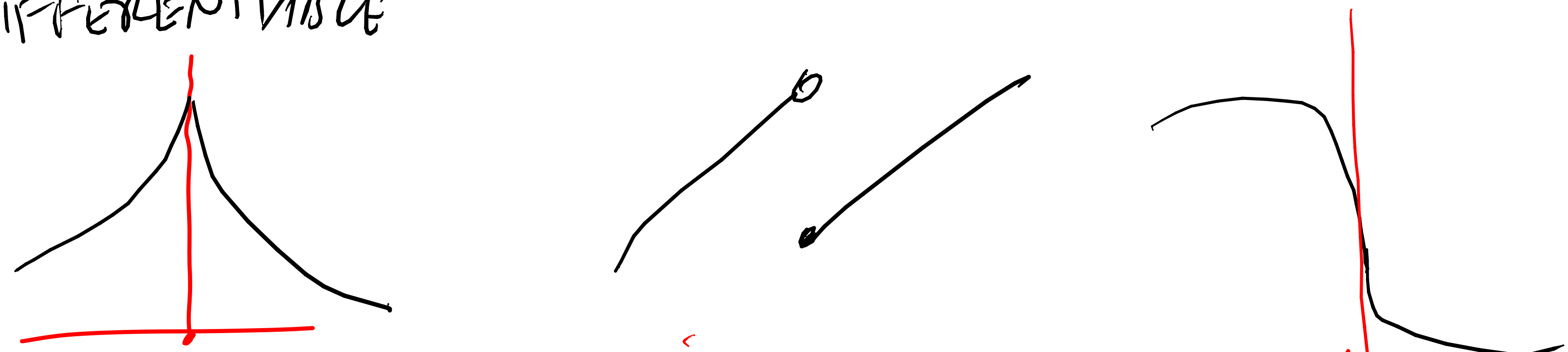
$$f(x)-f(a) = \underbrace{\frac{f(x)-f(a)}{x-a}}_{\downarrow \, f'(a)} \cdot \underbrace{(x-a)}_{\downarrow \, 0} \longrightarrow 0$$

as $x \to a$

so $f(x) \to f(a)$ as $x \to a$.

CONVERSE IS FALSE; $|x|$ is continuous at $a=0$ but deriv does not exist at 0

NOT DIFFERENTIABLE



Some ways: corner   jump   vertical tangent

$f(x)$, $f'(x)$

a function

$$f''(x) = [f'(x)]'$$

$$f'''(x) = (f'')'(x)$$

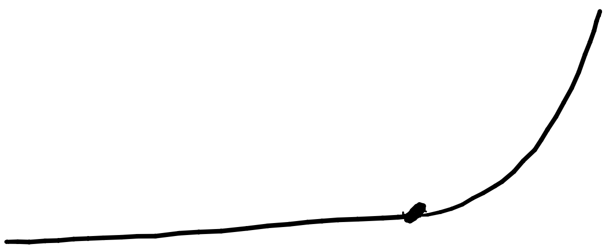
another notation is $f^{(3)}(x)$

Interpretation:

If $s(t)$ = distance as a function of time

then $s'(t)$ = velocity

$s''(t)$ = acceleration



TRAIN TRACKS

need acceleration to be continuous to stay on track

$$f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases} \qquad f' = \begin{cases} 0 & \text{if } x \le 0 \\ 2x & \text{if } x > 0 \end{cases} \qquad f'' = \begin{cases} 0 & \text{if } x < 0 \\ 2 & \text{if } x > 0 \end{cases}$$

CONTINUOUS

JUMP AT $X=0$