10/5/16 SECTION 2.5

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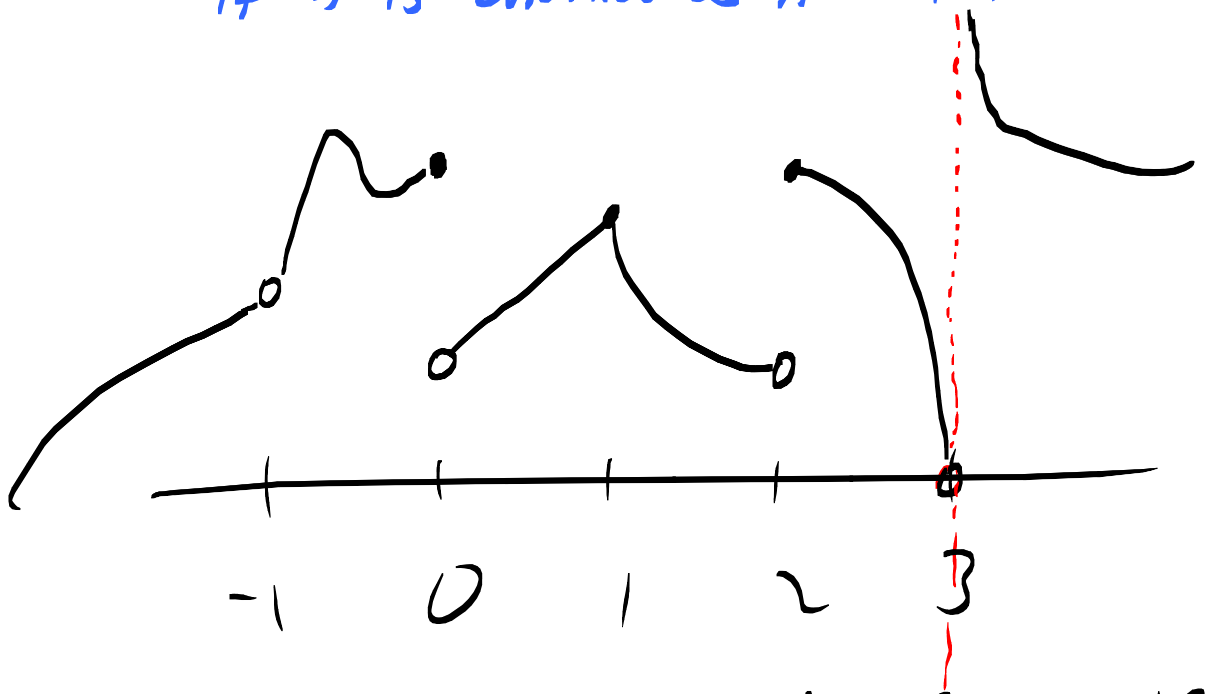

$\lim_{x \to a^+} f(x) = f(a)$

CONTINUOUS, CONTINUOUS ON LEFT, ON RIGHT, NEITHER:

1: CONT.   0:   -1:

2: CONT ON RT.   3:

SOME BOOKS: DISCONTINUOUS = NOT CONTINUOUS

$$f(x) = \frac{x^2 - x - 2}{x-2}$$

$\boxed{\text{CONVENTION: IF DOMAIN IS NOT SPECIFIED THEN THE DOMAIN IS ALL VALUES WHERE FORMULA MAKES SENSE.}}$

WHERE IS f CONTINUOUS?

WHY?

$$\frac{x^2 - x - 2}{x-2} = \frac{(x-2)(x+1)}{(x-2)} = x+1$$

when $x \neq 2$.

$$f(x) = \frac{x^2 - x - 2}{x-2}$$

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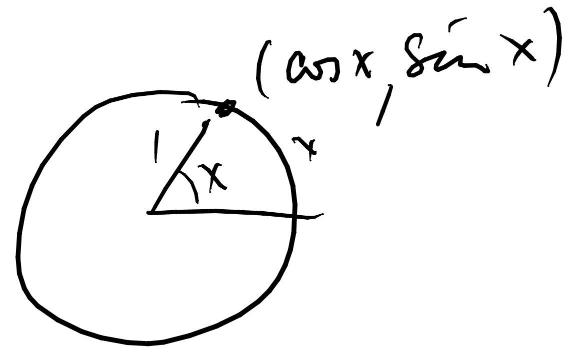
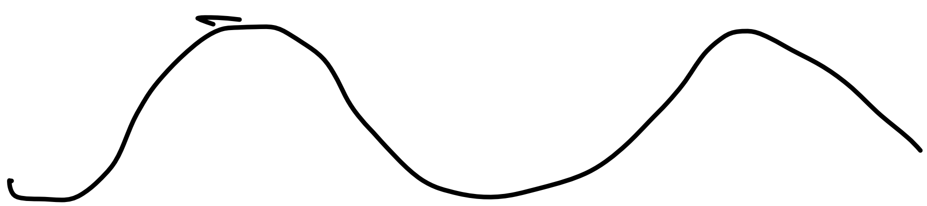
Is $\sin(x)$ CONTINUOUS?





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CONTINUOUS WHERE DEFINED:

- POLYNOMIALS — $3x^2 - 8x + 5$
- RATIONAL FUNCTIONS — $\dfrac{3x^2 - 8x + 5}{x^3 - x}$
- TRIG FUNCTIONS
- EXPONENTIAL FUNCTIONS

AND THEIR INVERSES:

ROOTS, INVERSE TRIG, LOGARITHMIC FUNCTIONS

$$(f \circ g)(x) = f(g(x))$$

If g is CONTINUOUS AT a AND
if f is CONTINUOUS AT $g(a)$ THEN
$f \circ g$ is CONTINUOUS AT a.

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$\sin(\ln(x))$ CONTINUOUS FOR $x > 0$.

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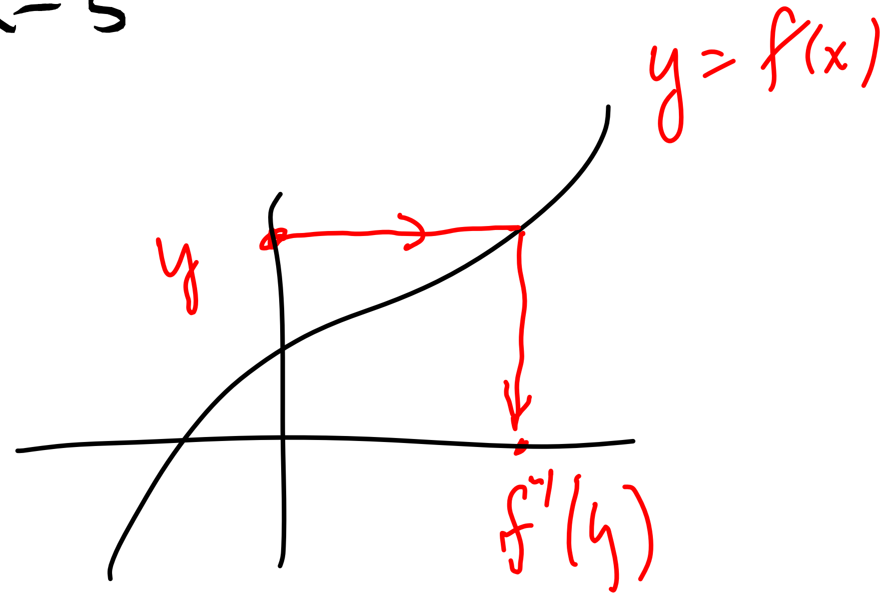
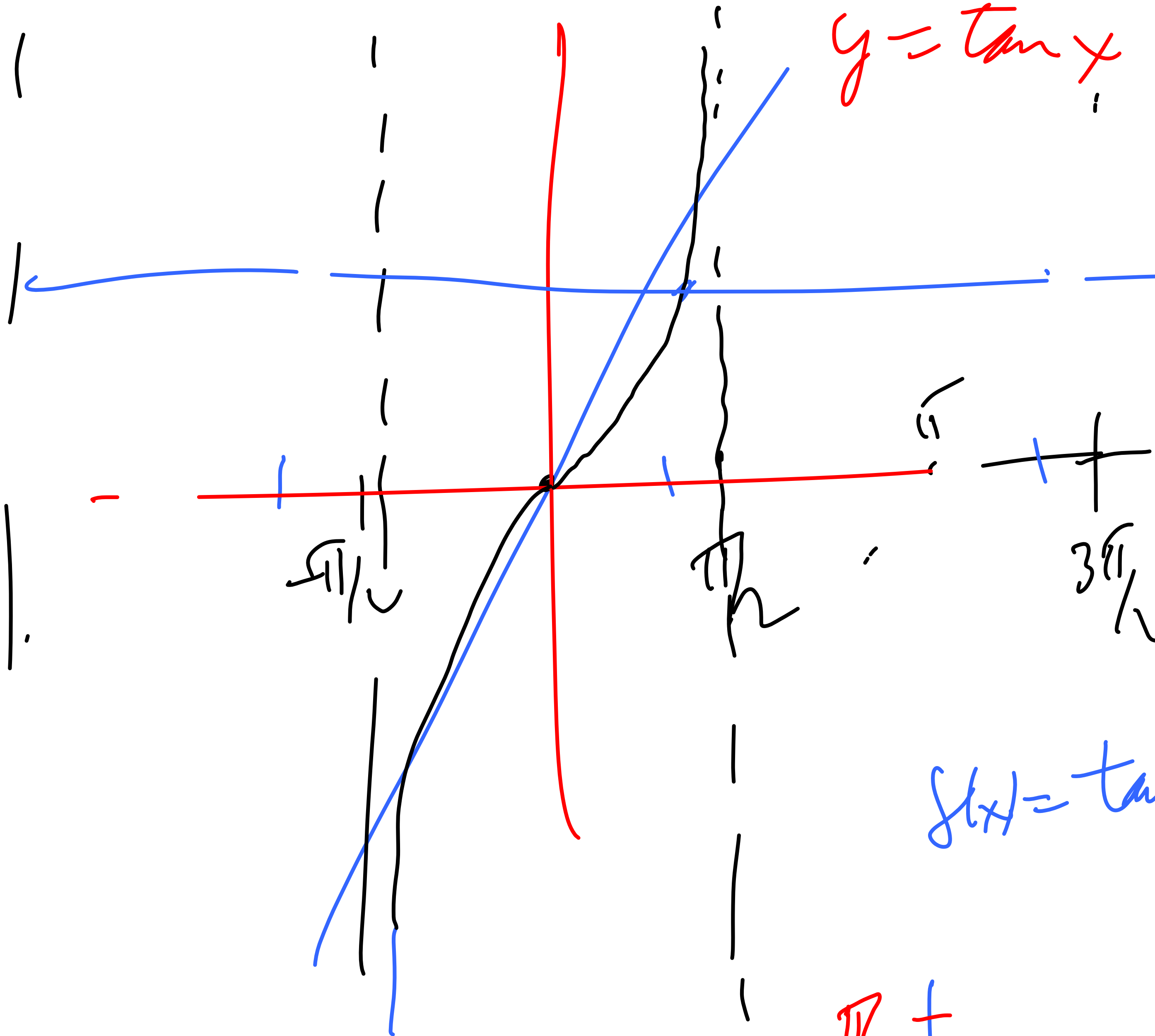
$$h(x) = \frac{\sin\left[\tan^{-1}(x) + \ln(x)\right]}{x-5}$$

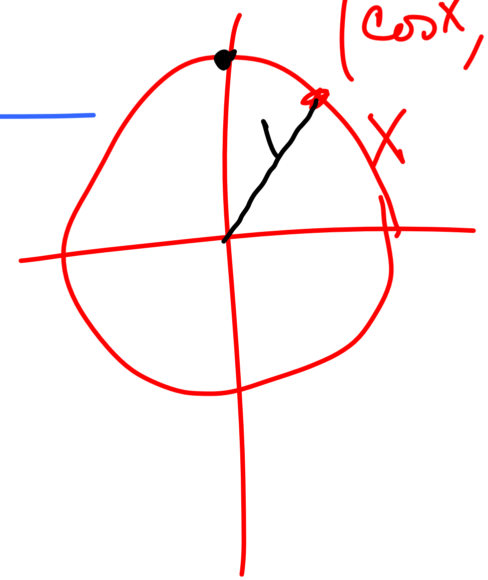
cont on $(0,5)$. and on $(5,\infty)$

Recall $\tan^{-1}x$:

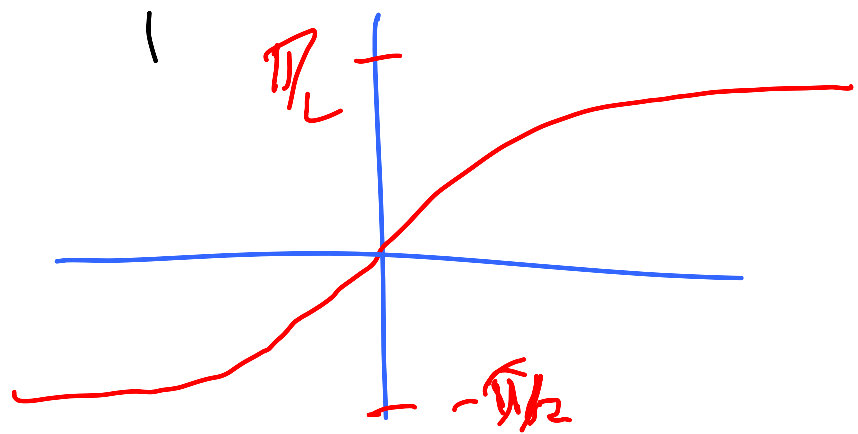


$y = \tan x = \dfrac{\sin x}{\cos x}$





$f(x) = \tan x$ $\quad -\pi/2 < x < \pi/2$



$\tan^{-1} x$

DOMAIN $= (-\infty, \infty)$

$$(f \circ g)(x) = f(g(x))$$

IF g IS CONTINUOUS AT a AND
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$\sin(\ln(x))$ CONTINUOUS FOR $x > 0$.

$$h(x) = \frac{\sin\left[\tan^{-1}(x) + \ln(x)\right]}{x-5}$$

$\ln(x)$ defined for $x > 0$
$\tan^{-1}(x)$ " " all x.
$x - 5 \neq 0$ except when $x = 5$
$\sin x$ CONTINUOUS FOR ALL x

h is CONTINUOUS ON $(0, 5)$ and $(5, \infty)$

(NOT INCLUDING ENDPOINTS)

IMPORTANT PROPERTY OF CONTINUOUS FUNCTIONS:

INTERMEDIATE VALUE THEOREM

SUPPOSE f IS CONTINUOUS ON $[a, b]$

SUPPOSE L IS A NUMBER BETWEEN $f(a)$ AND $f(b)$

THEN THERE EXISTS A NUMBER c WITH $a < c < b$

AND $f(c) = L$.

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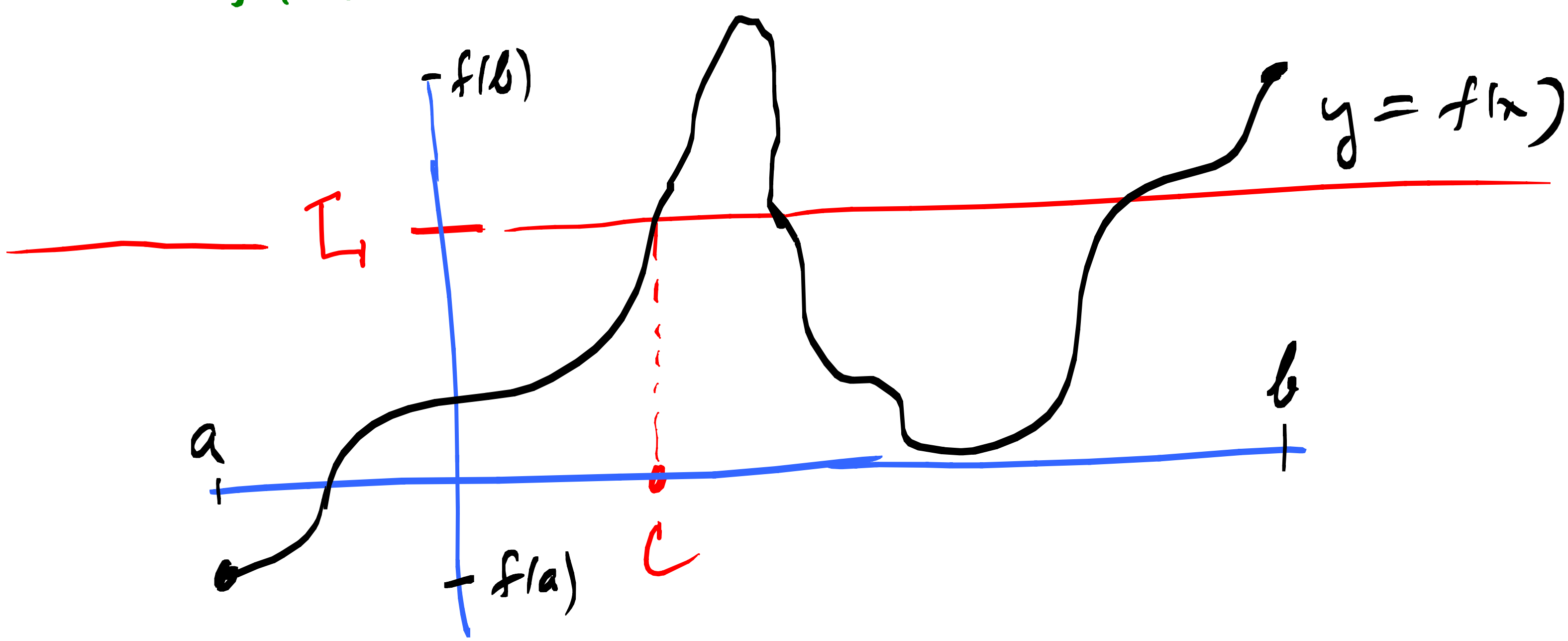

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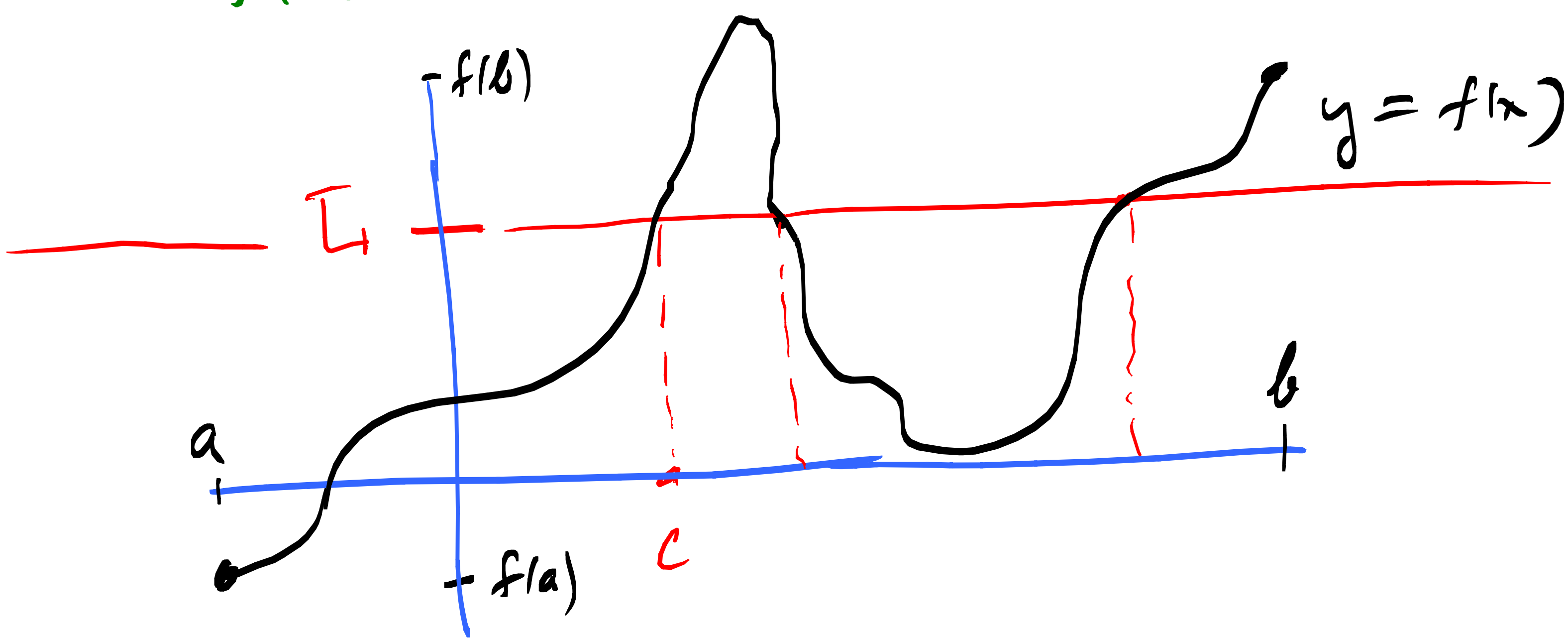

DOES NOT SAY HOW TO FIND c BUT IT IS USEFUL AS WE WILL SEE.

- THERE IS A NUMBER c WITH $0 < c < \pi/2$ AND $\tan(c) = 1.2$ WHY?

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$\tan 0 = 0 \quad \tan(\pi/2) = \infty$

$\tan(\frac{\pi}{2} - .0001)$ is LARGE – SO APPLY THEOREM

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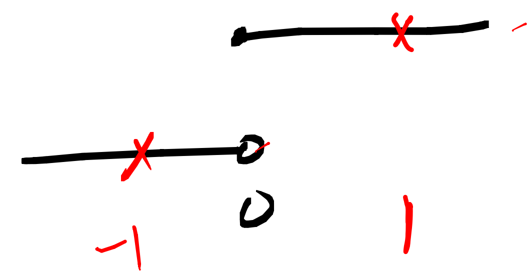
SAME AS: $\tan(b) - b = 2$

THEOREM DOES NOT APPLY TO DISCONTINUOUS FUNCTIONS

$H(x) =$ HEAVISIDE FUNCTION



$H(-1) = 0$ $\qquad$ $H(1) = 1$

$a = -1$ $\qquad$ $b = 1$

BUT H NEVER EQUALS $3/4$ BETWEEN -1 & 1

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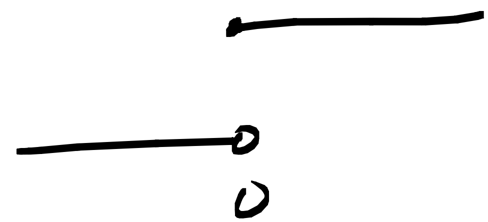

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FIND WHERE $f(x) = 0$ (AT LEAST APPROXIMATELY)

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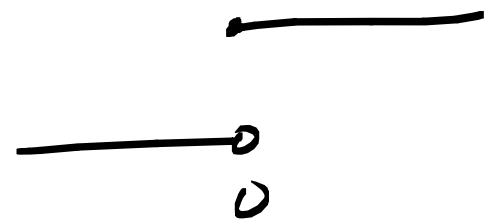

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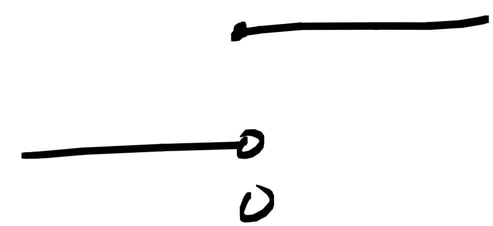

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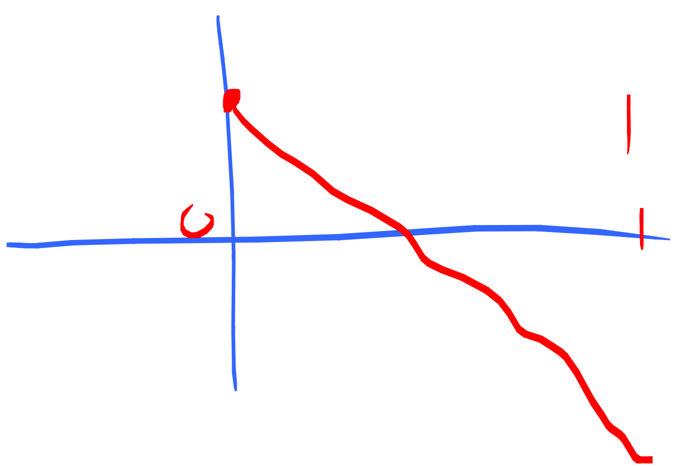

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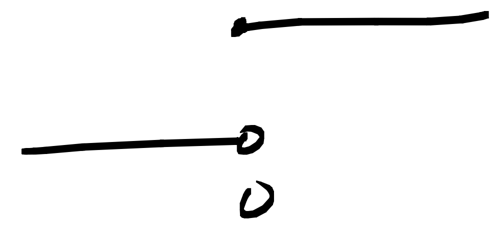

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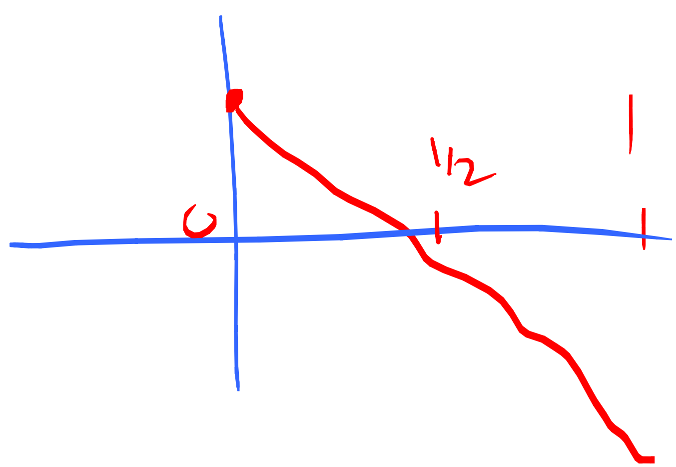
$$f(\tfrac{1}{2}) = \frac{3}{32} - \frac{4}{16} - 1 + 1 < 0$$



So $f = 0$ SOMEWHERE BETWEEN 0 & $1/2$