SECTION 2.3

LIMIT LAWS

SUPPOSE C IS A CONSTANT

SUPPOSE $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist.

SECTION 2.3

LIMIT LAWS

SUPPOSE c IS A CONSTANT

SUPPOSE $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist.

Then

(1) $$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

SECTION 2.3

LIMIT LAWS

SUPPOSE c IS A CONSTANT

SUPPOSE $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist.

Then

(1) (2) $\lim_{x\to a}\left[f(x) \pm g(x)\right] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

SECTION 2.3

LIMIT LAWS

SUPPOSE c IS A CONSTANT

SUPPOSE $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist.

Then

① $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

② $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

③ $\lim_{x \to a} [c f(x)] = c \lim_{x \to a} f(x)$

SECTION 2.3

LIMIT LAWS

SUPPOSE c IS A CONSTANT

SUPPOSE $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist.

Then

(1) (2) $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

(3) $\lim_{x \to a} [c f(x)] = c \lim_{x \to a} f(x)$

(4) $\lim_{x \to a} [f(x) g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

(5) $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ PROVIDED $\lim_{x \to a} g(x) \neq 0$

SECTION 2.3

LIMIT LAWS

SUPPOSE c IS A CONSTANT

SUPPOSE $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ exist.

Then

① ② $$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

③ $$\lim_{x\to a}[c f(x)] = c \lim_{x\to a} f(x)$$

④ $$\lim_{x\to a}[f(x) g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

⑤ $$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$$ PROVIDED $\lim\limits_{x\to a} g(x) \neq 0$

IDEA: BUILD COMPLICATED FUNCTIONS FROM SIMPLER ONES

$$\lim_{x\to a} 1 = 1 \qquad \lim_{x\to a} x = a$$

So $$\lim_{x\to a} x^2 = a^2 \qquad \lim_{x\to a} 3x^2 = 3a^2 \qquad \lim_{x\to a} 3x^2+5 = 3a^2+5$$

$$\lim_{x\to 3} \frac{x^2-2}{x-2} = 7$$

$$\lim_{x\to 2} \frac{x^2-2}{x-2} = ?$$

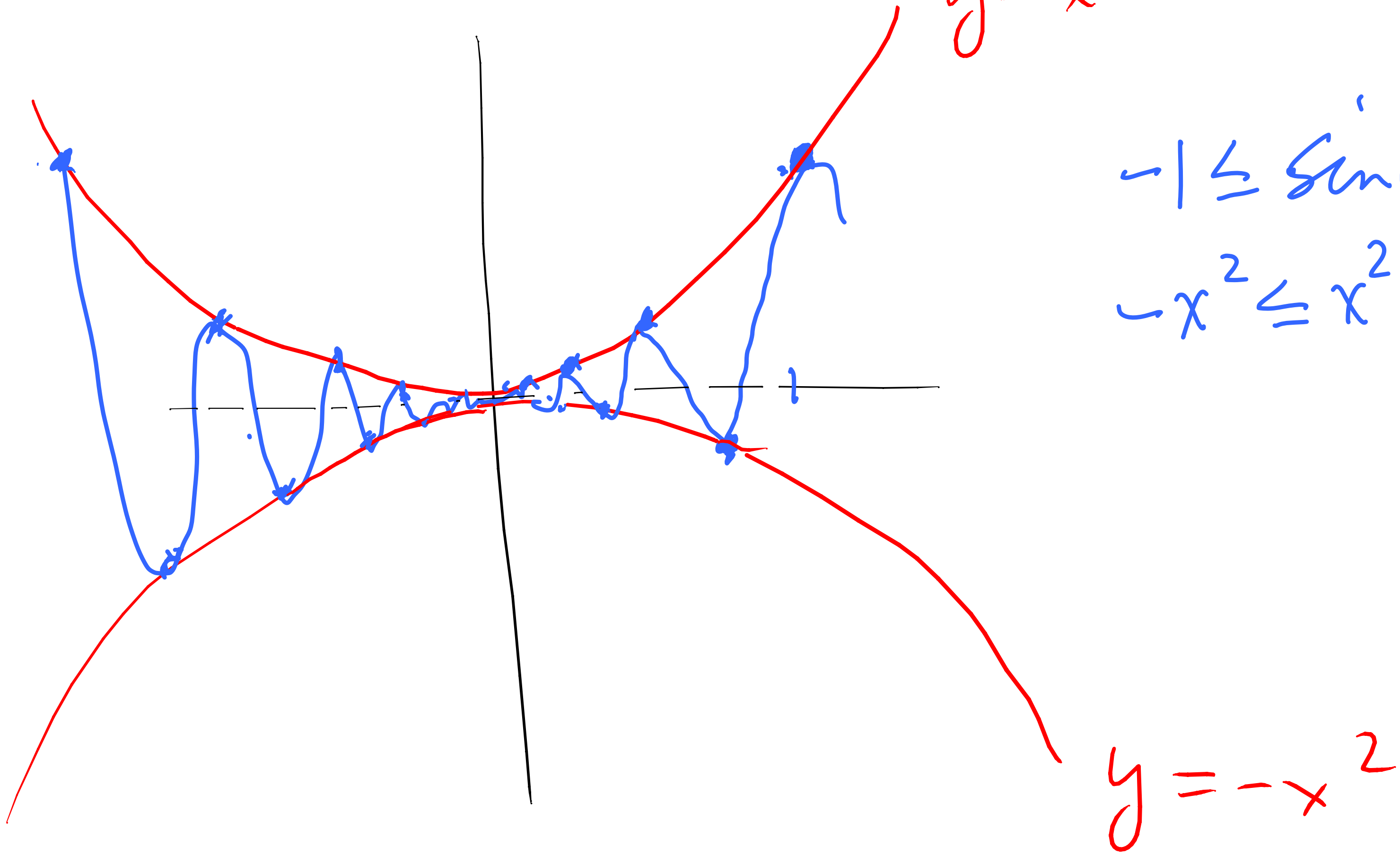
IF $f(x) = \dfrac{p(x)}{g(x)}$ p, g polynomials with $g(a) \neq 0$

THEN $\displaystyle\lim_{x\to a} f(x) = \frac{p(a)}{g(a)}$

$$\lim_{x\to 3} \frac{\sqrt{x} - 2x^3 + 1}{(x+1)(x-1)} =$$

$$\lim_{x\to 1} \frac{x^2-4x+5}{x^2-3x+2} =$$

How about $x^2 \sin(\frac{1}{x})$?

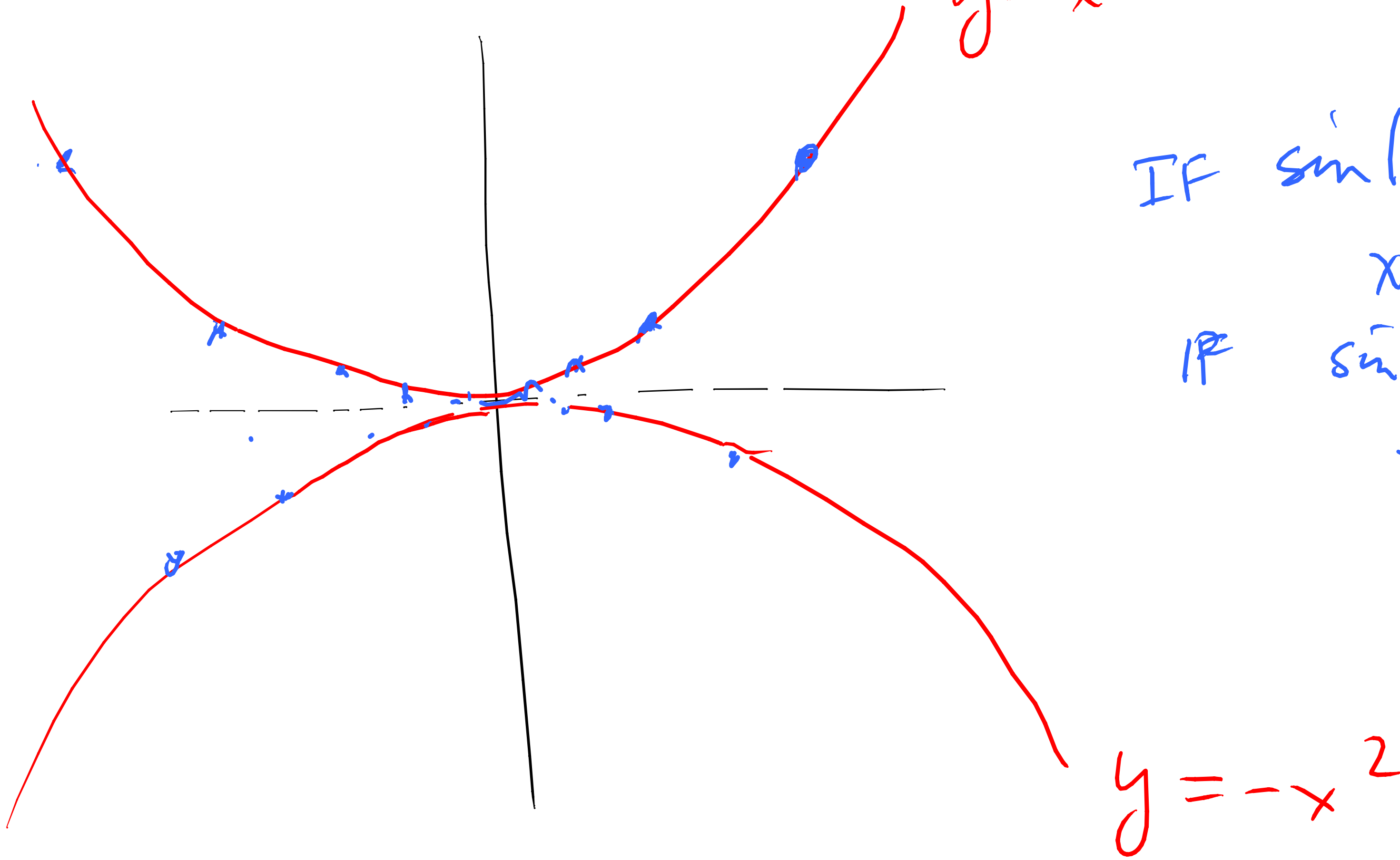


$$-1 \le \sin(\tfrac{1}{x}) \le 1$$

$$-x^2 \le x^2 \sin(\tfrac{1}{x}) \le x^2$$

$$\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0.$$

How about $x^2 \sin(\frac{1}{x})$?



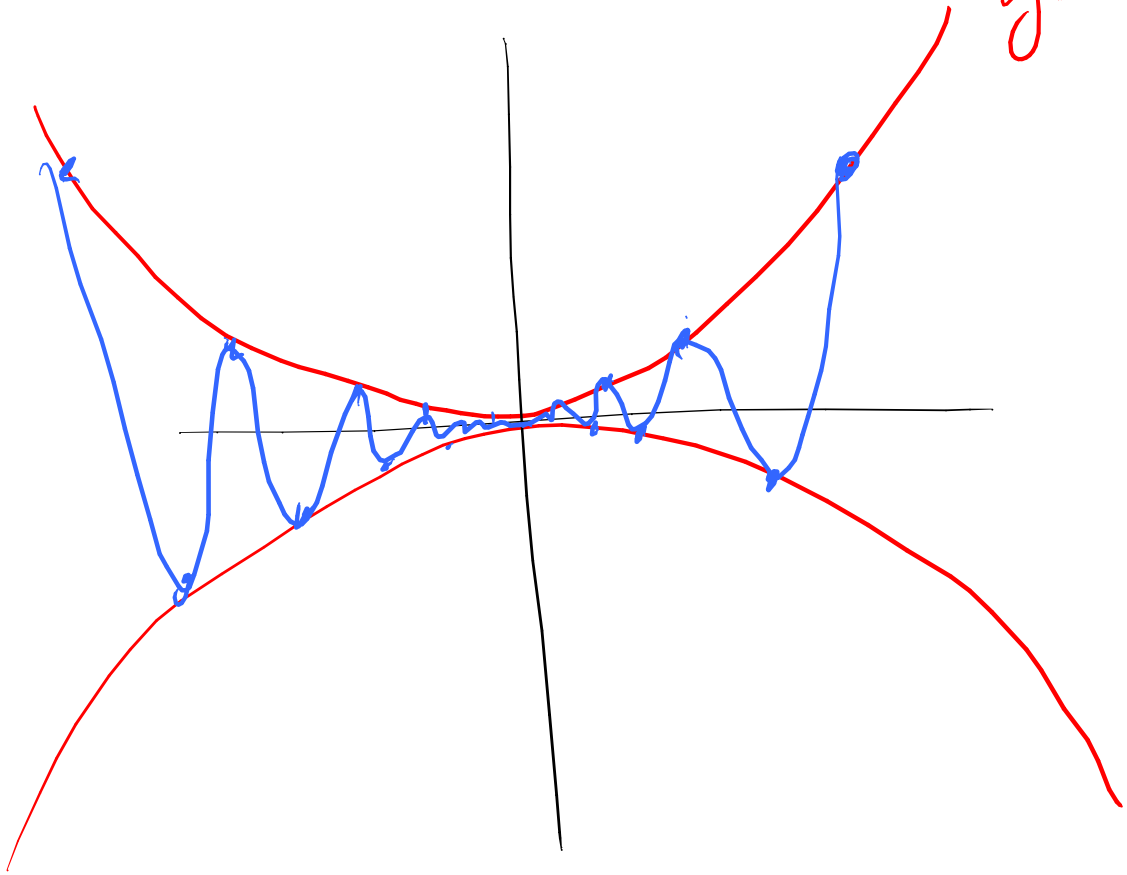
IF $\sin(\frac{1}{x}) = 1$ THEN

$x^2 \sin\frac{1}{x} = x^2$

IF $\sin(\frac{1}{x}) = -1$ THEN

$x^2 \sin\frac{1}{x} = -x^2$

How about $x^2\sin(\frac{1}{x})$?



IF $\sin(\frac{1}{x}) = 1$ THEN

$$x^2\sin\frac{1}{x} = x^2$$

IF $\sin(\frac{1}{x}) = -1$ THEN

$$x^2\sin\frac{1}{x} = -x^2$$

$$-1 \le \sin\left(\frac{1}{x}\right) \le 1$$

so $-x^2 \le x^2\sin\left(\frac{1}{x}\right) \le x^2$

IDEA $-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$

BOTH x^2 ; $-x^2 \longrightarrow 0$ as $x \longrightarrow 0$

SO $x^2 \sin\left(\frac{1}{x}\right) \longrightarrow 0$ as $x \longrightarrow 0$ "SQUEEZED"

IDEA $$-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$$

BOTH x^2 ; $-x^2 \to 0$ as $x \to 0$

SO $x^2 \sin\left(\frac{1}{x}\right) \to 0$ as $x \to 0$ "SQUEEZED"

SQUEEZE THEOREM:

If $g(x) \le f(x) \le h(x)$ WHEN x IS NEAR a,

AND IF $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$

Then $\lim_{x \to a} f(x) = L$

IDEA

$$-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$$

BOTH x^2 & $-x^2 \longrightarrow 0$ as $x \longrightarrow 0$

SO $x^2 \sin\left(\frac{1}{x}\right) \longrightarrow 0$ as $x \to 0$ "SQUEEZED"

SQUEEZE THEOREM:

If $g(x) \le f(x) \le h(x)$ WHEN x is NEAR a

AND IF $\lim\limits_{x \to a} g(x) = \lim\limits_{x \to a} h(x) = L$

Then $\lim\limits_{x \to a} f(x) = L$

× MOM

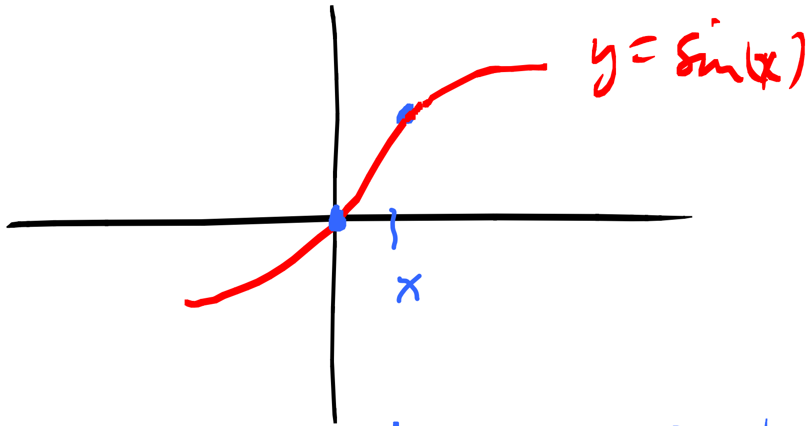
• KID

× DAD

• GOAL

Q: WHAT IS THE SLOPE OF $y = \sin(x)$ at $x = 0$?

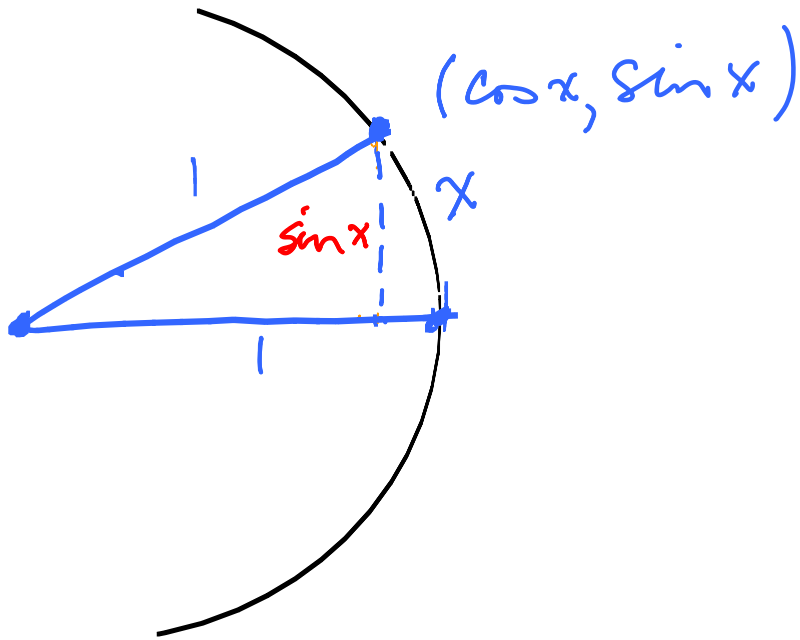
GEOMETRIC:



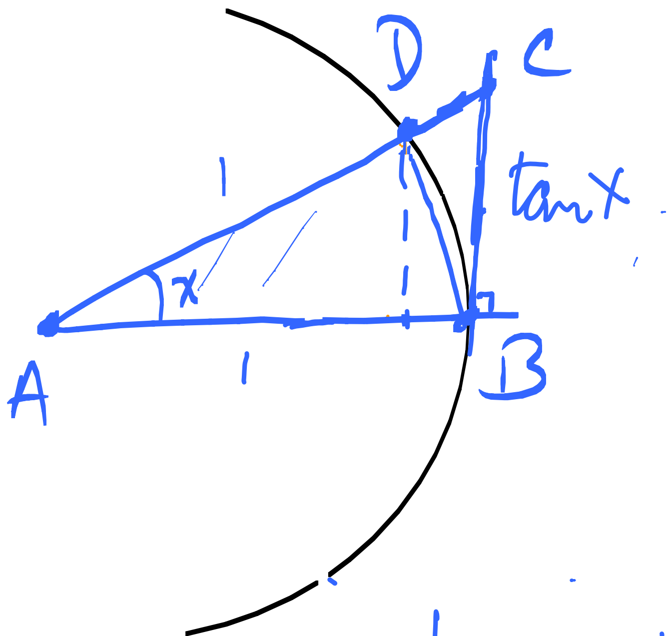
APPEARS SLOPE IS 1, CAN WE PROVE IT?

$$\lim_{x \to 0} \frac{\sin x - \sin 0}{x - 0} = \lim_{x \to 0} \frac{\sin x}{x} = ?$$

THEOREM: $\lim_{x \to 0} \frac{\sin x}{x} = 1$



THEOREM: $\lim_{x \to 0} \frac{\sin x}{x} = 1$



$Area(\Delta ABD) = \frac{1}{2} \sin x$

$Area(\Delta ADC) = \frac{1}{2} \tan x$

Area of sector $ABD = \frac{x}{2\pi} \cdot \pi = x/2$

$$\frac{1}{2} \sin x \le \frac{x}{2} \le \frac{1}{2} \tan x$$

$\sin x \le x.$

$x \le \tan x = \frac{\sin x}{\cos x}$

$\frac{\sin x}{x} \le 1$

$x \cos x \le \sin x$

$\cos x \le \frac{\sin x}{x}$

$\lim_{x \to 0} 1 = 1$ $\quad \lim_{x \to 0} \cos x = 1$

So $\lim_{x \to 0} \frac{\sin x}{x} = 1$ by SQ. THM.

APPLICATION!

$$\frac{\cos x - 1}{x} = \frac{(\cos x - 1)}{x} \cdot \frac{\cos x + 1}{\cos x + 1} = \frac{\cos^2 x - 1}{x(\cos x + 1)}$$

$$= -\left[\frac{(\sin x)}{x}\right]\left[\frac{(\sin x)}{(\cos x + 1)}\right] \xrightarrow[\text{AS } x \to 0]{} \frac{-1 \cdot 0}{2} = 0$$

COROLLARY:

SLOPE OF $y = \cos x$ at $x = 0$ is $= 0$.

$$\lim_{x \to 0} \frac{\sqrt[3]{x} - 5x^2 + 5 - 1/\sqrt{x+1}}{x + 7}$$

APPLICATION!

$$\frac{\cos x - 1}{x} = \frac{(\cos x - 1)}{x} \cdot \frac{\cos x + 1}{\cos x + 1} = \frac{\cos^2 x - 1}{x\,(\cos x + 1)}$$

$$= -\frac{(\sin x)}{x}\frac{(\sin x)}{(\cos x + 1)} \xrightarrow[\text{AS } x \to 0]{} \frac{-1 \cdot 0}{2} = 0$$

SLOPE OF $y = \cos x$ at $x = 0$ is $= 0$.