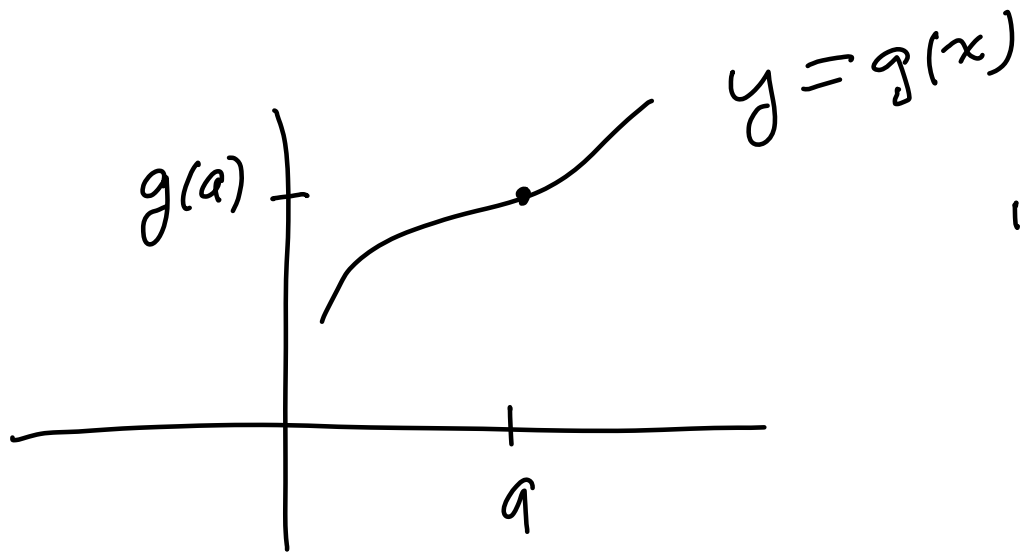


MATH 124 #2 9/30/16  
READING. QUESTIONS?

$\lim_{x \rightarrow a} f(x) = L$  MEANS: AS  $x$  APPROACHES  $a$  ( $x \neq a$ )  
THE VALUES  $f(x)$  APPROACHES  $L$

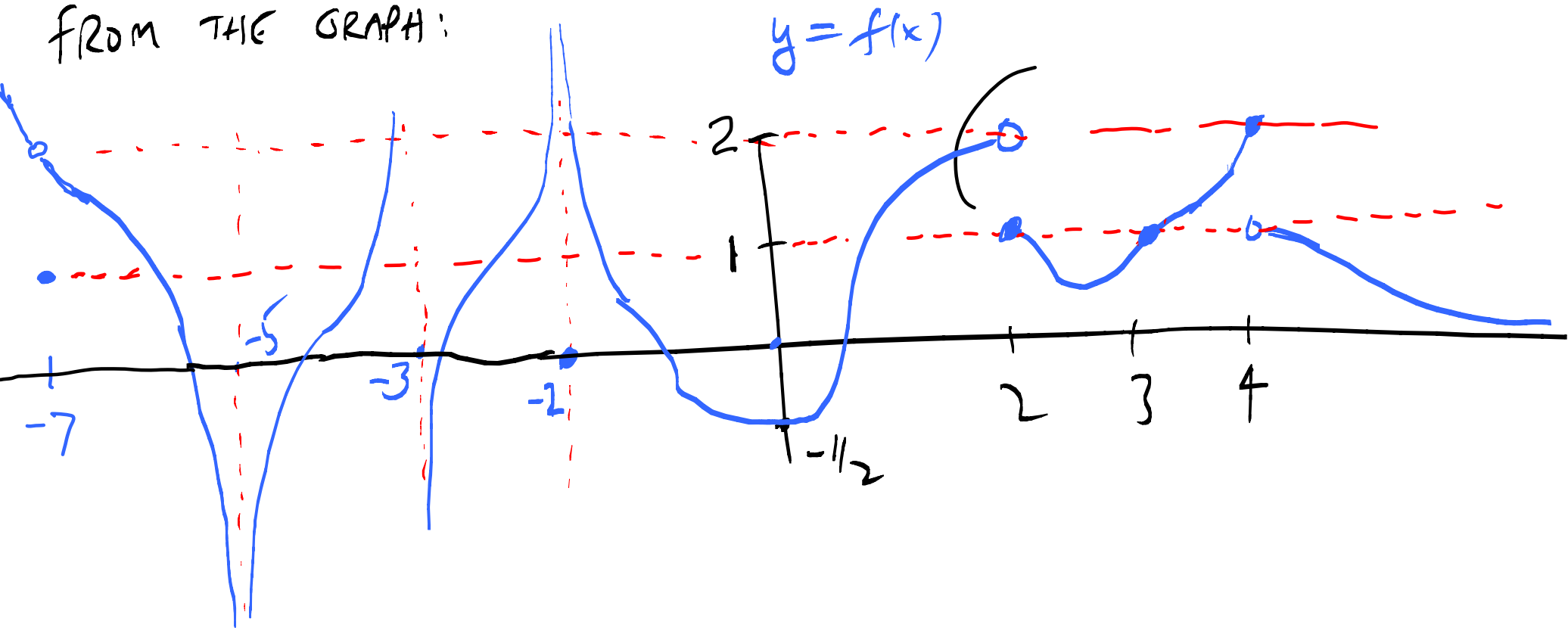
"limit"



"NICE"  $f$  HAS

$$f(a) = \lim_{x \rightarrow a} f(x)$$

FROM THE GRAPH:



$$\lim_{x \rightarrow 3} f(x) = 1$$

$$\lim_{x \rightarrow 2^+} f(x) = 1$$

$$\lim_{x \rightarrow 2^-} f(x) = 2$$

$$f(-7) = 1$$

$$\lim_{x \rightarrow 0} f(x) = -\infty$$

$$\lim_{x \rightarrow -2} f(x) = \infty$$

$$\lim_{x \rightarrow -3} f(x) = \text{D.N.E.}$$

$$\lim_{x \rightarrow -7} f(x) = 2$$

$$\lim_{x \rightarrow -5} f(x) = -\infty$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

$$f(-2) = 0$$

$$\lim_{x \rightarrow 0} \frac{x^2}{x}$$

$\frac{x^2}{x}$  is not defined at  $x=0$

$$\lim_{x \rightarrow 0} \frac{x^2}{x}$$

$\frac{x^2}{x}$  is not defined at  $x=0$

$$\frac{x^2}{x} = x$$

(true where both are defined)

$$\text{So } \lim_{x \rightarrow 0} \frac{x^2}{x} = \lim_{x \rightarrow 0} x = 0$$

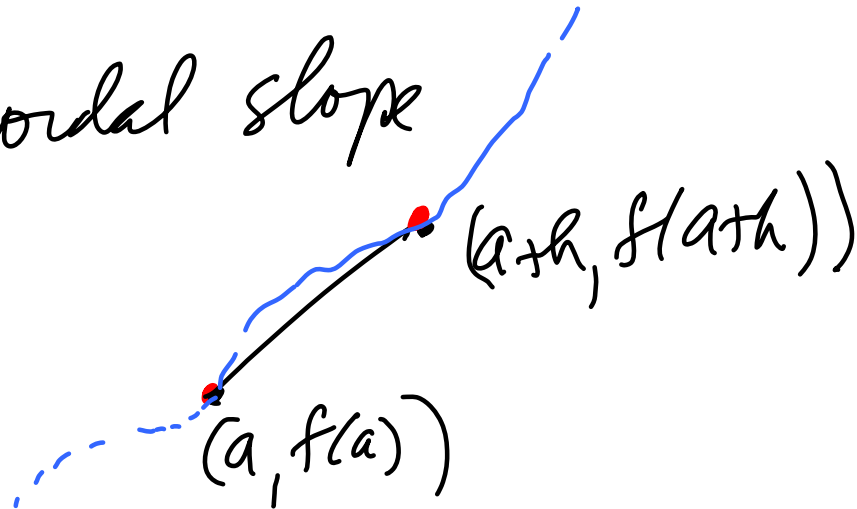
$$\lim_{x \rightarrow 0} \frac{x^2}{x}$$

$\frac{x^2}{x}$  is not defined at  $x=0$

$$\frac{x^2}{x} = x \quad (\text{true where both are defined})$$

$$\therefore \lim_{x \rightarrow 0} \frac{x^2}{x} = \lim_{x \rightarrow 0} x = 0$$

Chordal slope



$$\lim_{x \rightarrow 0} \frac{x^2}{x}$$

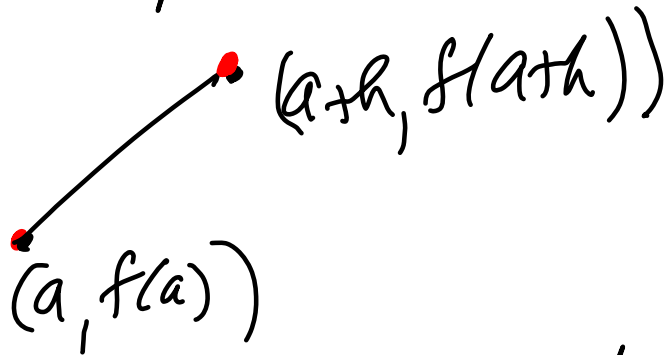
$\frac{x^2}{x}$  is not defined at  $x=0$

$$\frac{x^2}{x} = x \quad (\text{true where both are defined})$$

$$\therefore \lim_{x \rightarrow 0} \frac{x^2}{x} = \lim_{x \rightarrow 0} x = 0$$

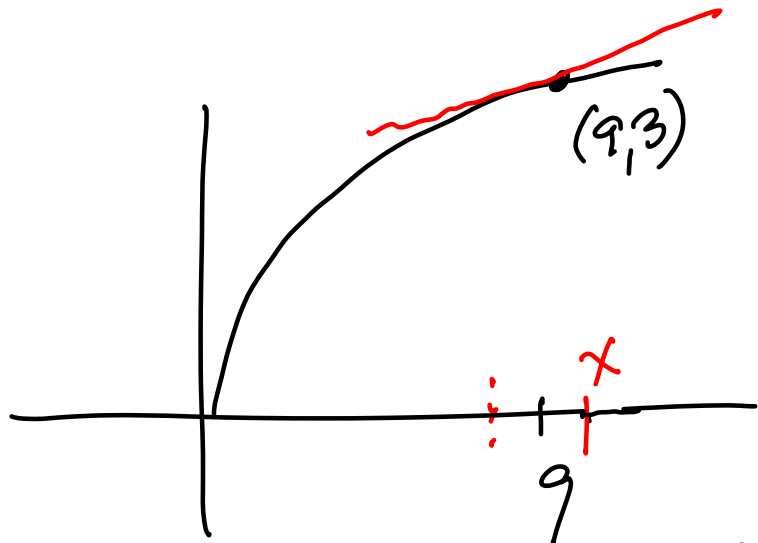
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Chordal slope =  $\frac{f(a+h) - f(a)}{a+h - a}$



$$= \frac{f(a+h) - f(a)}{h}$$

Not defined at  $h=0$  but for "nice"  $f$   
we can find  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$



$$y = \sqrt{x}$$

Find eq. of tangent  
line when  $x = 9$

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Slope at  $x = 9$  is

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$$

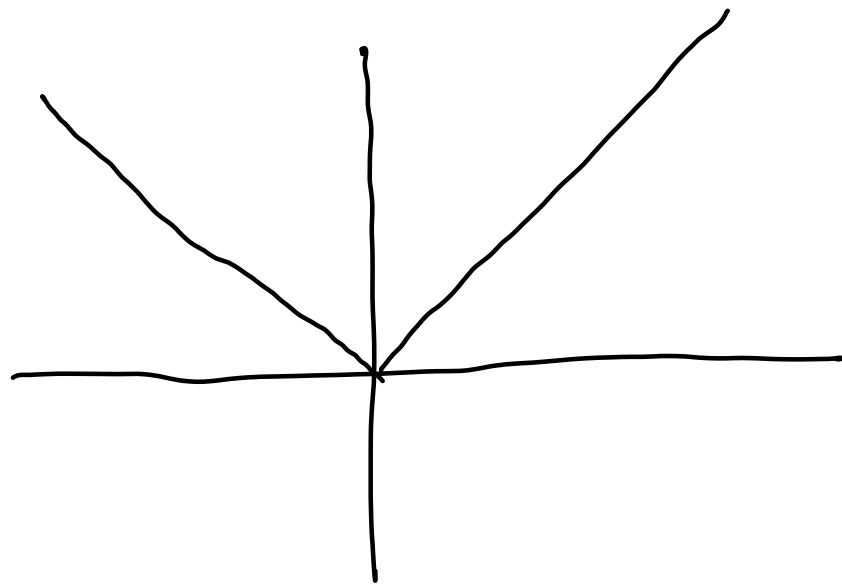
$$\frac{\sqrt{x} - 3}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3} = \frac{x - 9}{(x - 9)(\sqrt{x} + 3)} \stackrel{x \neq 9}{=} \frac{1}{\sqrt{x} + 3}$$

$\rightarrow \frac{1}{6}$  as  $x \rightarrow 9$ .

$$y = \frac{1}{6}(x - 9) + 3$$

TANGENT LINE  
AT  $x = 9$

Graph  $y = |x|$



$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\frac{y(x) - y(0)}{x - 0} = \frac{|x|}{x}$$

$$\lim_{x \rightarrow 0} \frac{|x|}{x} = \text{DNE}$$

Find slope at  $x = 0$ .

SLOPE

0 1

\_\_\_\_\_ 0 \_\_\_\_\_

\_\_\_\_\_ 0 \_\_\_\_\_

-1



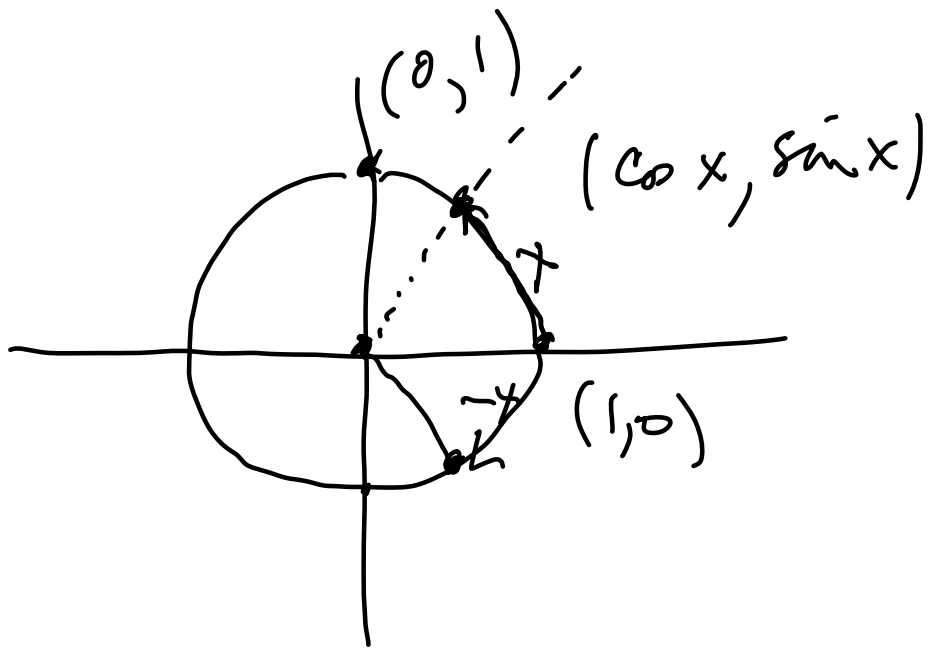
$$\lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3}$$

$$\frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \frac{\cancel{(x-9)} (\sqrt{x}+3)}{\cancel{x-9}} = \sqrt{x}+3$$

$\uparrow$   
 $x \neq 9$

$\downarrow$  as  $x \rightarrow 9$   
 $\in$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\cos x} = 0$$



$$\sin\left(-\frac{\pi}{2}\right) = -1$$

$$\cos\left(-\frac{\pi}{2}\right) = 0$$

$$|\sin x| \leq 1$$

