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Correction to page 367 of Harmonic Measure.

C. J. Bishop noticed recently that the counting argument just after (2.15) is incomplete. With thanks, we replace it here by an argument due to Bishop that will appear in his forthcoming book "Fractals in Analysis and Probability" with Y. Peres.

On page 367 replace the 8 lines beginning "We first prove" by the following:

We first prove the right-hand inequality of (2.14), that there exists a curve $\Gamma \supset E$ with $\Lambda_1(\Gamma) \leq c_2\beta^2(E)$. As in the proof of Theorem 2.1 we construct the rectangles S_I and write L_I for the longer side of S_I and $\beta_I L_I$ for its shorter side. If S_0 and S_1 are the two immediate descendents of a rectangle $S = S_I$, then

$$L_0 + L_1 \leq L + c_3\beta_I^2 L_I. \tag{$*$}$$

In Case 1 (*) follows from (2.8) and in Case 2 (*) is trivial. For $I = (i_1, i_2, \ldots, i_n), i_j = 0, 1$ define

$$E_I = \bigcap\{S_J \cap A_J : J = (i_1, i_2, ..., i_m), m \leq n\}.$$

Then $E \subset \bigcup\{E_I : |I| = n\}$, $E_I^o \cap E_J^o = \emptyset$ if $|I| = |J|$ and $I \neq J$, and

$$L_I \leq \text{diam}(E_I) \leq \text{diam}(S_I) \leq \sqrt{2}L_I$$

because E_I meets each side of S_I. It then follows from the decay rate for $\text{diam}(S_I)$ that

$$\text{diam}(E_J) \leq \frac{1}{2}\text{diam}(E_I)$$

if $|J| \geq |I| + 48$ and I is an initial segment of J. Also, since E_I meets every side of S_I,

$$\text{Area}(E_I) \geq c_4\text{Area}(S_I) = c_4\beta_I L_I^2.$$

For any dyadic cube Q define

$$\mathcal{E}(Q) = \{E_I : E_I \cap Q \neq \emptyset,\ \text{diam}(E_I) \leq \ell(Q) \leq 2\text{diam}(E_I)\}.$$

Then the union $\bigcup\{E_I : E_I \subset \mathcal{E}(Q)\}$ falls inside the narrowest strip containing $E \cap 3Q$ and covers Area almost every point of $E \cap 3Q$ at most 48 times. Hence

$$\sum_{\mathcal{E}(Q)} \text{Area}(E_I) \leq c_5\beta_E(3Q)(\ell(Q))^2$$

so that

$$\sum_{\mathcal{E}(Q)} \beta_I \leq c_6\beta_E(3Q). \tag{$**$}$$

Now let $R_n = \sum_{|I|=n} L_I$. Then by (*) and induction

$$R_n \leq c_7\text{diam}(E) + c_8\sum_Q\sum_{\mathcal{E}(Q)}\beta_I^2 L_I.$$

On the other hand since $\beta_I \geq 0$,

$$\sum_{\mathcal{E}(Q)}\beta_I^2 \leq \Big(\sum_{\mathcal{E}(Q)}\beta_I\Big)^2 \leq c_6^2\beta_E^2(3Q)$$

by (**). Therefore

$$R_n \leq c_7\text{diam}(E) + c_9\sum_Q\beta_E^2(3Q)\ell(Q) \leq C\beta^2(E).$$

Now continue on page 367 from the phrase "To estimate the lengths ...."