This is a self diagnostic exam is designed to test your preparedness for

Math 318, A Second Course in Linear Algebra.

Please review the content of Math 308 and then take this exam in your own time to measure your ability to succeed in Math 318.

Purpose of this test:
This Math 318 course will assume thorough knowledge of linear algebra from Math 308. Lack of comfort with that material will lead to many struggles in this course. There is no time limit for this exam, but if you know the required background material, the test should not take you more than an hour. If you find many of the questions challenging, then this Math 318 is not the course for you.

The following questions will all refer to the following linear system of equations.

$$\begin{cases} 2x_1 + 5x_2 + x_3 = 8 \\ - 3x_2 - x_3 = -2 \\ 2x_1 + 14x_2 + 4x_3 = 14 \end{cases}$$

1. Find a matrix A and a vector $\mathbf{b}$ that expresses the above linear system in the form $A\mathbf{x} = \mathbf{b}$.
2. Find all solutions to the system of equations. Then write down one explicit solution to the system.
3. Are the columns of A linearly independent? Justify your answer.
4. Do the columns of A span $\mathbb{R}^3$? If not, find equations for the plane or line that they span.
5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation associated to the matrix A. That is, $T(\mathbf{x}) = \mathbf{Ax}$ for all vectors $\mathbf{x} \in \mathbb{R}^{\mathbf{3}}$. Determine (with reason) if T is 1-1 and onto.
6. Find a basis for the following subspaces: (i) $\text{Ker}(T)$ — the kernel of T, (ii) $\text{Range}(T)$ — the range of T, and (iii) $\text{Col}(A)$ — the column span of A.
7. What is $\text{nullity}(A)$? Use this number to find $\text{rank}(A)$. Which quantity in the previous question should agree with $\text{rank}(A)$? Why?
8. Compute $\det(A)$. (Hint: based on some earlier problems you should be able to know the answer!)
9. Let B be an invertible 3×3 matrix and consider the linear transformation, $S : \mathbb{R}^3 \to \mathbb{R}^3$ given by $S(\mathbf{x}) = BAB^{-1}\mathbf{x}$. Determine wether or not S is an invertible linear transformation. (Hint: Based on earlier problems this can also be answered quickly!)
10. Find all eigenvalues and bases for all associated eigenspaces of A. (Hint: You already found one of them!)
11. Is T diagonalizable? If no, why not? If yes, give matrices P and D that put A into diagonal form.