Math 208 M Spring 2022 Final exam

NAME (First,Last) : ....

Student ID ....

UW email ....

- Please use the same name that appears in Canvas.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.
- Write your NAME (first, last) on top of every ODD page of this exam.
- If you run out of space, continue your work on the back of the last page and indicate clearly on the problem page that you have done so.
- Unless stated otherwise, you **MUST** show your work and justify your answers.
- Please be precise. Imprecise language such as "'this matrix is linearly independent"' or "'this matrix is one to one"' will be marked down.
- Your work needs to be neat and legible.
- This exam contains 4 pages and 6 problems, please make sure you have a complete exam.

Problem 1 Read both parts of this problem, before you start doing calculations. You are given the vectors $v_1 = (1, a, -1)$, $v_2 = (-1, 1, 2)$, $v_3 = (4, -4, a)$.

1. Find all values of a such that v_1, v_2, v_3 do not span R^3.

2. Give an example of a and b such that the vector $(0, 2, b)$ is not in the span of v_1, v_2, v_3. Show your work to explain how you found your example.

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Problem 2 Suppose that the general solution in vector form of $A\vec{x} = 0$ is $(-x_4, x_4, 2x_4, x_4)$. Answer the following questions; **remember to justify your answers**. If you think that you do not have enough information to answer a question, just answer "'not enough information"'.

1. How many columns does A have ?

2. How many rows does A have ?

3. Find a basis for Null(A), the nullspace of A

4. What is the rank of A ?

5. Is the first column of A in the span of the other columns of A?

Problem 3 Given $W = \{(x, y, z) \text{ in } R^3 : x + y + z = 0\}$

Find a 3x3 matrix A such that the null space of A is equal to W or explain why this is not possible.

Find a 3x3 matrix A such that det (A)=1 and the column space of A is equal to W or explain why this is not possible.

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Problem 4 Given $A = \begin{pmatrix} 2 & 5 \\ 5 & 2 \end{pmatrix}$

1. Find all eigenvalues of A.

2. For each eigenvalue λ you found, give a basis B for $E(\lambda)$ (the eigenspace of λ).

3. Diagonalize A, that is find matrices P and D with $A = PDP^{-1}$.

4. The columns of P form a basis B_1 for R^2. Find $[(1,0)]_{B_1}$, the vector of coordinates of (1,0) with respect to B_1.

5. Find $[A \begin{pmatrix} 1 \\ 0 \end{pmatrix}]_{B_1}$ (Hint: can you use D ?)

Problem 5 Consider the linear transformation $T : R^3 \to R^2$ defined by $T(v) = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} v$

1. is T one to one ? Justify your answer.

2. Is it possible to find a linear transformation $S : R^2 \to R^3$ $S(v) = Bv$ for some matrix B, such that $T \circ S$, the composition of S and T, (recall that this means that $T \circ S(v) = T(S(v))$) is one to one ? If you think it is possible, find B, and show your work to explain how you found B. If you think this is not possible, justify why.

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Problem 6 Suppose A is a 3×3 matrix with characteristic polynomial $p(\lambda) = -\lambda(\lambda - 2)^2$ and that the eigenspace for eigenvalue $\lambda = 2$ is $E_2 = \{(x, y, z) \text{ in } R^3 : x + y + z = 0\}$

1. Is A invertible ? Justify your answer.

2. Is A diagonalizable ? Justify your answer.

3. What is the rank of A ? Justify your answer.