Your Name

Your Signature

Student ID #

Section (Tues.)	1:30	2:30	3:30
(circle one)	PA	PB	PC

- Please show all of your work, including your steps in row reduction. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Do not use scratch paper–put all of your work on the exam sheets. If you need more room, use the backs of the pages and indicate that you have done so.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- Raise your hand if you have a question.
- Be careful!! Double-check to make sure you copy the numbers correctly and you don't make careless arithmetic errors.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	8	
2	8	
3	6	
4	5	
5	8	
6	8	
7	8	
8	10	
9	9	
10	10	
Total	80	

1. (8 points) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 5 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$.

(a) Determine whether the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent or linearly independent.

(b) Find the dimension of $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Briefly explain.

2. (8 points) (a) Show that the vector $\mathbf{x} = \begin{bmatrix} -5 \\ 3 \\ 1 \\ 1 \end{bmatrix}$ is in the nullspace of the matrix $A = \begin{bmatrix} 2 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

(b) Find a basis for $\text{null}(A)$ containing $\mathbf{x}$. Briefly explain.

3. (6 points) (a) A vector $\mathbf{x}$ in $\mathbb{R}^2$ has coordinate vector $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ with respect to the basis $\mathscr{B} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$. Find the coordinates of $\mathbf{x}$ with respect to the standard basis.

(b) Find the coordinates of the vector $\mathbf{x} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ with respect to the basis $\mathscr{B}$.

4. (5 points) Solve for the matrix X. Assume that all matrices are $n \times n$ and invertible as needed.

$$AX(D+BX)^{-1} = C$$

5. (8 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$ for a 2×2 matrix A. Suppose you know that $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Find A.

6. (8 points) Let $\mathbf{u}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 3 \\ 1 \\ -3 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} -4 \\ -1 \\ 5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 13 \\ 5 \\ -11 \end{bmatrix}$.

   Determine whether $\operatorname{span}\{\mathbf{u}_1, \mathbf{u}_2\} = \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2\}$. Explain your reasoning.

7. (8 points) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$. Find all eigenvalues and eigenvectors for A.

8. (10 points) (a) Find <u>all</u> eigenvalues and eigenvectors for the matrix $A = \begin{bmatrix} 2 & 2 & -3 \\ 0 & 0 & 3 \\ 0 & 2 & -1 \end{bmatrix}$.

(b) Is A diagonalizable? If yes, write the diagonalization. If no, explain why not.

9. (9 points) For each of the following, give an example or state that it is not possible. You do not have to explain why. 3 points each.

   (a) A 2×4 matrix A with $\text{nullity}(A) = 2$ and $\text{col}(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -6 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$.

   (b) A 4×4 matrix A with $\det(A) = 0$ and $\text{rank}(A) = 3$.

   (c) A one-to-one linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$.

10. (10 points) True or False questions, 2 points each. Just write your answer True or False. No explanation required, no partial credit.

(a) If A is a $n \times n$ matrix and A is singular, then A^2 is singular.

True or False:

(b) Let A be a 3×3 matrix whose eigenvalues are 4 with algebraic multiplicity 1 and -1 with algebraic multiplicity 2. Then A is invertible.

True or False:

(c) If A is a $n \times n$ matrix, then $\text{row}(A) = \text{col}(A)$.

True or False:

(d) There is a 3×3 matrix A with real entries whose eigenvalues are 1, $2+i$, $3-i$.

True or False:

(e) If a system of linear equations has four equations and seven variables, then it has infinitely many solutions.

True or False: