Final Exam
Math 308A Autumn 2018
Dami Lee

Name:______________________________________

- One handwritten 8.5 by 11 sheet of notes is allowed. 2-sided is OK.

- The only calculator allowed is the Texas Instruments TI-30X IIS.

- Please try to fit your answer in the space provided. If you need more space, the instructor will provide you with additional paper.

- Show your work for full credit.

- Cheating will not be tolerated. Any instances will be reported to the department of Community Standards and Student Conduct and no credit will be given for the exam in question.

1. (1 point each) Determine whether each of the following statements is (T)rue or (F)alse.

   (a) If $\mathbf{A}$ is a 3×5 matrix, then $\text{nullity}(\mathbf{A}) \leq 3$. T F

   (b) If $\mathbf{A}$ is a square matrix, then $\text{row}(\mathbf{A}) = \text{col}(\mathbf{A})$ T F

   (c) If $\mathbf{u}_1, \mathbf{u}_2$ are eigenvectors of $\mathbf{A}$, then $\mathbf{u}_1 + \mathbf{u}_2$ is an eigenvector of $\mathbf{A}$. T F

   (d) If $\mathbf{A}$ is diagonalizable, then $\mathbf{A}$ is invertible. T F

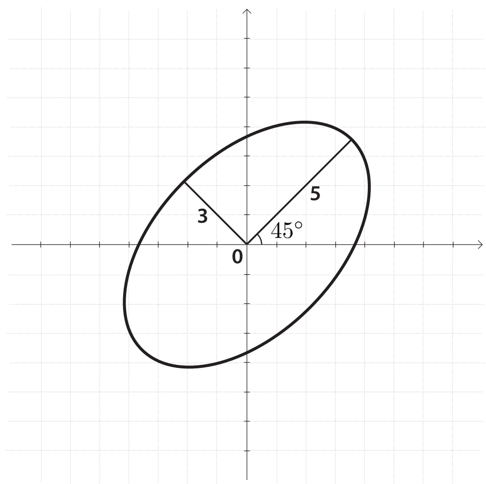
   (3 points each) For each of the following statements, determine whether it is (T)rue, (F)alse, or (S)ometimes true/sometimes false. Provide an example (or two, if the answer is (S)) to justify your answer.

   (e) A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is one-to-one. T F S

   (f) A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^4$ is onto. T F S

2. (10 points) Find all vectors in $\mathbb{R}^3$ that are *not* in the span of $\left\{ \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} \right\}$. Justify your answer.

3. (a) (6 points) Find the associated matrix $\mathbf{A}$ to a linear transformation $T_{\mathbf{A}} : \mathbb{R}^2 \to \mathbb{R}^2$ that maps the unit circle ($x^2 + y^2 = 1$) to the given ellipse. (Use $\cos 45° = \sin 45° = \frac{\sqrt{2}}{2}$ if needed.)



(b) (3 points) Is such a linear transformation unique?

(c) (3 points) Determine the area of the given ellipse.

4. (a) (4 points) Find a basis for the given subspace $\mathbf{S}$ by deleting linearly dependent vectors.

$$\mathbf{S} = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} \right\}$$

(b) (2 points) What is the dimension of $\mathbf{S}$?

(c) (6 points) Find a matrix whose null space is **S**.

5. (8 points) Find a 3 by 3 matrix whose eigenvalues are $\lambda = 2$, 0, and -1, and the corresponding eigenvectors are $\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$, respectively. Write the answer as a single matrix.

6. (8 points) Compute the determinant of

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 & 1 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}^3 \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 5 \end{bmatrix}^{-1}$$

7. (10 points) Let $\mathbf{A}$ be any 2 by 2 stochastic matrix. Is $\mathbf{A}$ always diagonalizable? Justify your answer.