Spring 2018 MATH 307 Midterm 2
80 pts total

Name: _____

Instruction:

- Nothing but writing utensils and a double side 4in $\times$ 6in notecard are allowed.
- Unless otherwise specified, you must show work to receive full credit.

1 (24pts). Find the solution to the initial value problem

$$y'' - y = xe^x, \qquad y(0) = 1, \quad y'(0) = -1/4$$

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(Extra space just in case you need it.)

2 (12pts). Consider the linear homogeneous equation

$$t^2y'' - 4ty' + 6y = 0, \quad t > 0.$$

[a] Find all values of p such that $y(t) = t^p$ is a solution to the above equation.
[b] Find the general solution to the differential equation.

3. Hanging a mass of $m = 1\ kg$ stretches the spring by $5/(2\pi^2)$ meters. Use $g = 10\ m/s^2$ as the acceleration due to gravity.

[a] (4pts) Find the spring constant k.

[b] (4pts) Denote by $y(t)$ the displacement the mass at any time t, with $y(t) > 0$ when the spring is stretched from the equilibrium position and $y(t) < 0$ when the spring is compressed. Write down the differential equation that governs the motion of this undamped mass-spring system. *Note: you need to put in actual numbers, not just symbols, for the mass, spring constant, and etc., to get full credit. No need to show work.*

[c] (16pts) Suppose the initial displacement is $-\frac{1}{4\sqrt{2}}$ and the initial velocity is $\frac{\pi}{2\sqrt{2}}$. Find $y(t)$ for all t and express the answer in the form of $y(t) = A\cos(\omega t - \phi)$.
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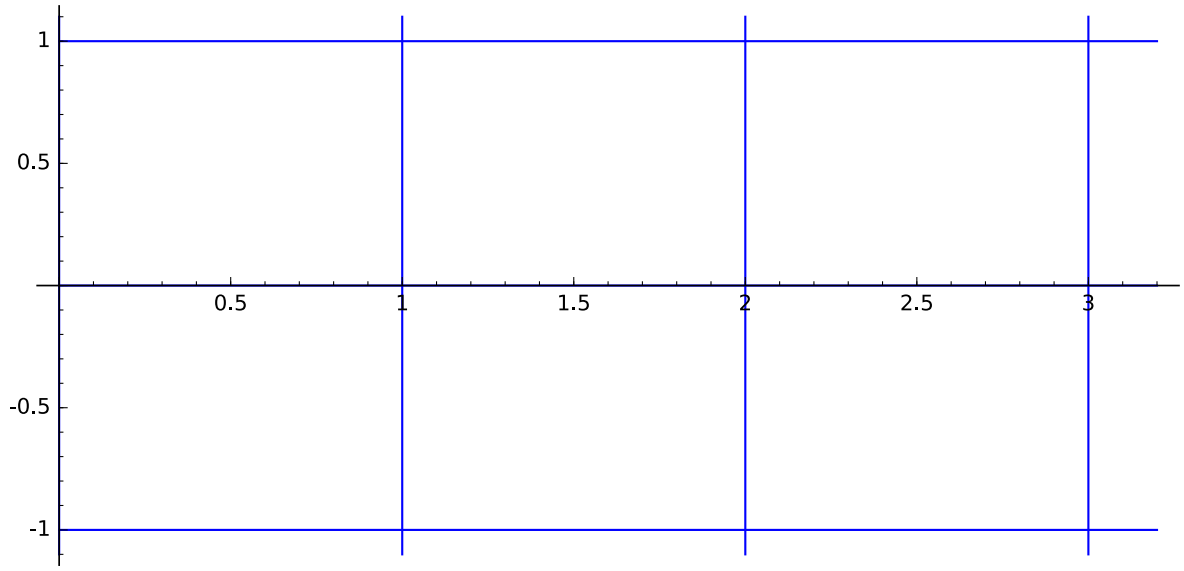
3. (Continued)

[d] (6pts) Graph the solution (be sure your graph illustrates the period, amplitude, and phase shift accurately). *You don't have to explain.*



4 (14pts). You have a spring in a damping media, but you don't know the spring constant nor the damping constant. To find that out, you decide to attach a mass of 1 kg to the spring and plot the motion of this unforced damped spring-mass system. The graph below is a plot of the displacement of the mass at any time t. Write down the differential equation governing its motion.

Explain your reasoning to get full credit. Note: you should write down actual (estimated) numbers based on what you gather from the graph, not just a symbolic equation.

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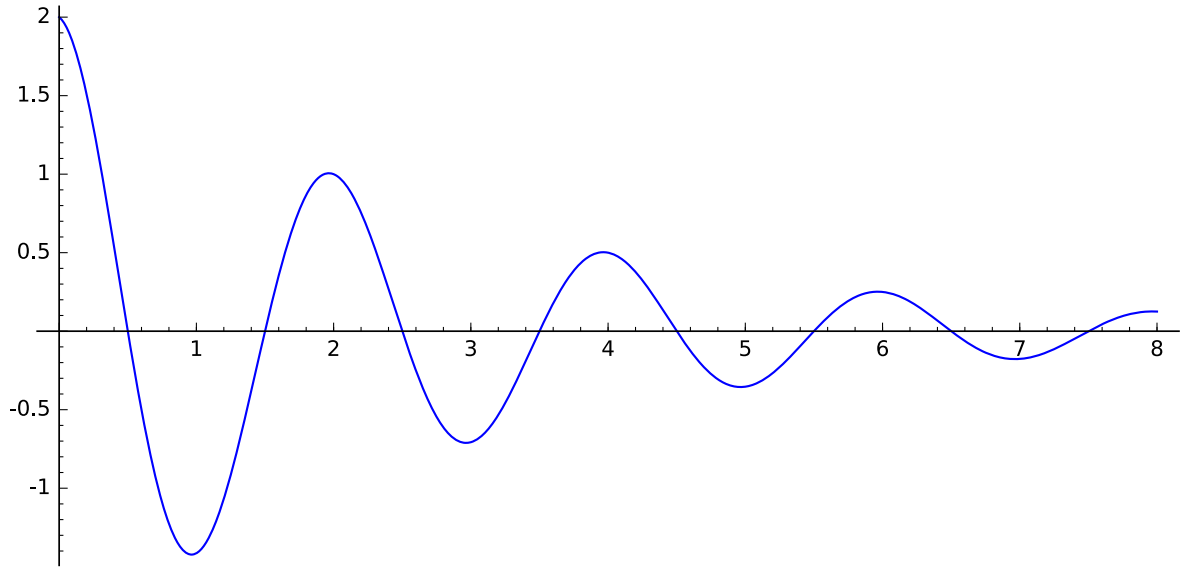

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