Math 307M **Midterm 2A** **Winter 2018**

Your Name

Your Signature

Section (circle one) MA MB MC

Problem	Total Points	Score
1	20	
2	20	
3	15	
4	15	
5	30	
Total	100	

- This exam is closed book. No Notes. If you forget a formula, ask one of us.
- No graphing or symbolic calculators are allowed. You may not use cell phones during the exam.
- Show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around $\boxed{\text{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate that you have done so.
- If you are not sure what a question means, raise your hand and ask us.
- The hints are suggestions only.

Formulas

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\cos\theta\cos\phi = \frac{1}{2}\left(\cos(\theta+\phi) + \cos(\theta-\phi)\right)$$
$$\sin\theta\sin\phi = \frac{1}{2}\left(\cos(\theta-\phi) - \cos(\theta+\phi)\right)$$
$$\sin\theta\cos\phi = \frac{1}{2}\left(\sin(\theta+\phi) + \sin(\theta-\phi)\right)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$
$$\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$
$$\cosh\theta = \frac{e^{\theta}+e^{-\theta}}{2}$$
$$\sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$$

1 (20 points)

Find the solution to

$$y'' + 6y' + 13y = 0$$
$$y(0) = 1 \qquad y'(0) = 1$$

and write it in amplitude-phase form: $y(t) = Ae^{rt}\cos(\omega t - \phi)$.

2 (20 points)

Find the general solution to

$$y'' - 4y' + 4y = 2t^2 + 1 + e^t$$

3 (15 points)

For linear differential equations of the form

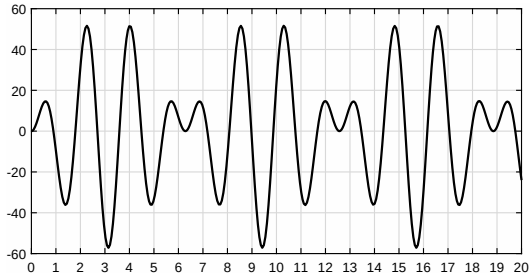
$$t^2y'' + aty' + by = 0$$

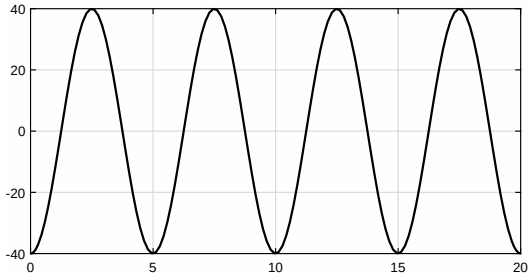
we look for solutions of the form $y(t) = t^r$ instead of $y(t) = e^{rt}$. Find two solutions of the from $y(t) = t^r$ for the DE below and use them to solve the IVP

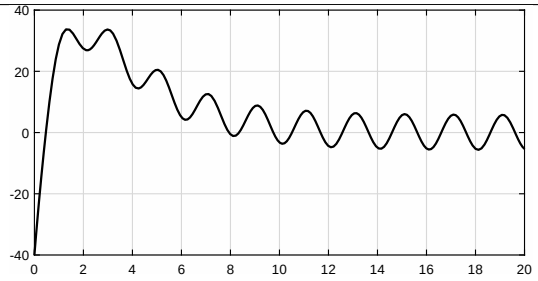
$$t^2y'' + ty' - 16y = 0$$
$$y(1) = 0 \qquad y'(1) = 1$$

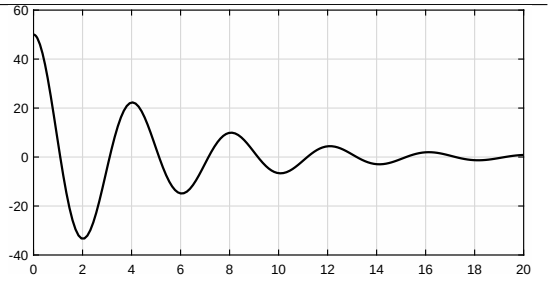
4 (15 points) An undamped mass spring system is released from equilibrium with a velocity of 8 m/s. The mass is 1 kg and it oscillates with an amplitude of 2 meters. There is no forcing. Find the spring constant k.

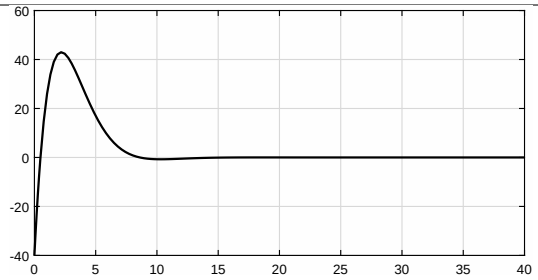
5 (30 points)

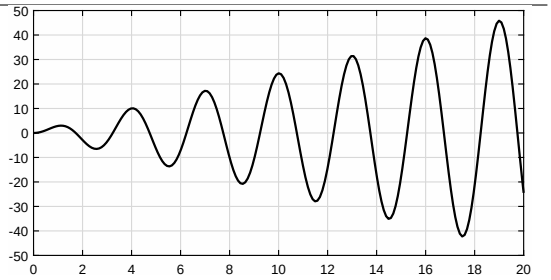
A 

B 

C 

D 

E 

F 

Which graph shows a solution of a forced damped mass-spring system? What is the forcing frequency?

Which graph is a solution to an unforced overdamped mass-spring system?

Which graph is a solution to an unforced underdamped mass-spring system? What is the natural quasi-frequency?

Which graph is a solution to a system that is unforced and undamped? What is the natural frequency?

Which two graphs show solutions to forced and undamped mass-spring systems?
What phenomena does each exhibit?
Write a plausible formula for each of these two solutions. *I don't care if you write sine when it shold be cosine, but try to get the basic form and the frequencies approximately right.don't worry about amplitude or phase.*