

MATH 307 F
EXAM 2
WINTER 2018

Name _____ Student ID # _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

1	8	
2	10	
3	12	
4	12	
5	8	
Total	50	

- Your exam should consist of this cover sheet, followed by 5 problems. Check that you have a complete exam.
- Pace yourself. You have 50 minutes to complete the exam and there are 5 problems across 6 pages. Try not to spend more than 10 minutes on each problem.
- Unless otherwise indicated, show all your work and justify your answers.
- Unless otherwise indicated, your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- You may use a scientific calculator and one 8.5×11-inch sheet of handwritten notes. All other electronic devices (including graphing and programmable calculators and calculators with calculus functions) are forbidden.
- The use of headphones or earbuds during the exam is not permitted.
- You are not allowed to use your phone for any reason during this exam. Turn your phone off and put it away for the duration of the exam.
- There may be multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- Raise your hand if you have any questions.
- Double check your work if you have time left.

GOOD LUCK!

1. (a) (6 points) Solve the initial value problem

$$4y'' + 7y' - 2y = 0, \quad y(0) = 6, \quad y'(0) = \beta$$

where β is a constant.

- (b) (2 points) Find β so that the solution approaches zero as $t \rightarrow \infty$.

2. Consider the second order linear differential equation $4t^2y'' + y = 0$ with $t > 0$.

(a) (2 points) Verify that $y_1(t) = t^{1/2}$ is a solution to the above differential equation.

(b) (7 points) Use **reduction of order** to find a second, linearly independent solution.
(Important: Although this is an Euler equation, no credit will be given unless you use reduction of order). **This topic is not covered in all sections.**

(c) (1 point) Write down the general solution to the above differential equation.

3. (a) (4 points) Write down the best guess for the form of the particular solution y_p . Use capital letters A, B, \dots etc as needed for the undetermined coefficients. You DO NOT need to find the values of A, B, \dots etc.

$$y'' + 4y = \cos(3t) - \sin(2t) + t^2 - 1$$

- (b) (8 points) Find a particular solution y_p to the following nonhomogeneous equation. There should be no arbitrary constants in your answer, i.e. you DO need to find the values of A, B, \dots etc.

$$y'' + 2y' = -2t + 1 + 4te^{2t}$$

4. Consider the following spring-mass systems.

- (a) (4 points) A mass of 20 grams is attached to a certain spring so that when the mass comes to rest at its equilibrium position the spring is stretched 4 cm beyond its natural length. The system is damped so that the damping force would have magnitude equal to 268 dyne (1 dyne = 1 gram · cm/s²) if the mass were moving at a speed of 8 cm/s.

Now the mass is lifted 5 cm above its equilibrium position and released with a downward velocity of 6 cm/s. Assume there is no external forcing function acting on the mass after this. Set up (but DO NOT SOLVE) the initial value problem for the equation of motion $u(t)$ of the mass. (Use 9.8 m/s² for the acceleration due to gravity but be careful with units!)

- (b) (4 points) A different spring-mass system which is underdamped has solution

$$u(t) = -\frac{3}{2}e^{-\frac{1}{2}t} \cos(3t) + \frac{2}{3}e^{-\frac{1}{2}t} \sin(3t).$$

Rewrite this solution in the form $Re^{\alpha t} \cos(\mu t - \delta)$ where R is a positive number. Keep four decimal places if you round any of these numbers.

- (c) (2 points) Draw a rough sketch of the graph of $u(t)$ using your answer from part (b). (Note that the initial conditions of the system are $u(0) = -\frac{3}{2} < 0$ and $u'(0) = \frac{11}{4} > 0$ which you may assume without computation. Your graph should include these features).



- (d) (2 points) What is the first time the mass in part (b) crosses through the equilibrium position? Keep at least four decimal places.

5. (a) (2 points) Briefly explain why the general solution y of $y'' + p(t)y' + q(t)y = g(t)$ is the sum of the general solution y_c of the corresponding homogeneous equation and any particular solution y_p of the nonhomogeneous equation. (No need to write an essay. Just convince me that you understand why this is true. A couple lines should suffice.)

- (b) (6 points) Find a differential equation that has

$$y = c_1 e^{2t} \cos(3t) + c_2 e^{2t} \sin(3t) + \frac{1}{2} \sin(3t)$$

as its general solution.