Name: ____________________

*Mathematics 207 J*
*University of Washington*

**February 2, 2022**

**MIDTERM 1**

Here are the rules:

- This exam is closed book. No note sheets, calculators, or electronic devices are allowed.
- In order to receive credit, you must **show all of your work**; to obtain full credit, you must provide mathematical justifications. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give numerical answers in exact form (for example $\ln(\frac{\pi}{3})$ or $5\sqrt{3}$ or $e^{2.5}$).
- Simplify $e^{a\ln(x)} = x^a$ for $x > 0$.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, plus a cover sheet. Please make sure that your exam is complete.
- You have 50 minutes to complete the exam.
- HAVE FUN!

| Problem | Possible | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 8 | |
| 3 | 10 | |
| 4 | 11 | |
| 5 | 16 | |
| Total | 55 | |

Good Luck!

**Problem** 1. Consider the initial value problem,

$$\frac{dy}{dt} + \frac{2y}{5+t} = 6, \qquad y(0) = 0.$$

(a) (2 points) Circle your answer

(i) Is this a *linear* differential equation? **YES** **NO**

(ii) Is this a *separable* differential equation? **YES** **NO**

(b) (8 points) Solve the initial value problem.

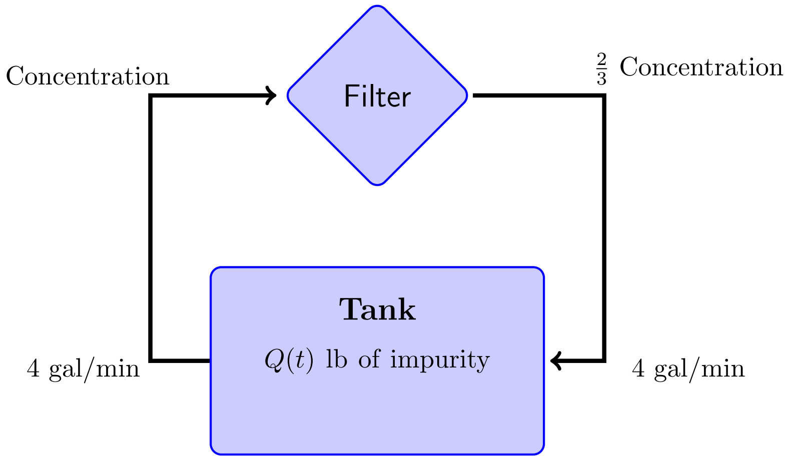
**Problem** 2. (8 points) You deposit \$10000 into a savings account at 5% annual interest, compounded continuously. You withdraw money from the account at a continuous rate of \$1000 per year. After how many years will the account balance be \$0?

**Problem** 3. (10 points) Consider the initial value problem

$$t^2 \frac{dy}{dt} = \frac{1}{y+3}, \qquad y(1) = -5.$$

Solve the initial value problem. Give an explicit formula for $y$.

**Problem** 4. A tank holding water that contains an impurity $Q$ is attached to a recirculating filter, as pictured below. The liquid passes through the filter at the rate of 4 gal/min. The filter removes 1/3 of the amount of $Q$ that passes through it, and lets the remaining 2/3 go back into the tank.



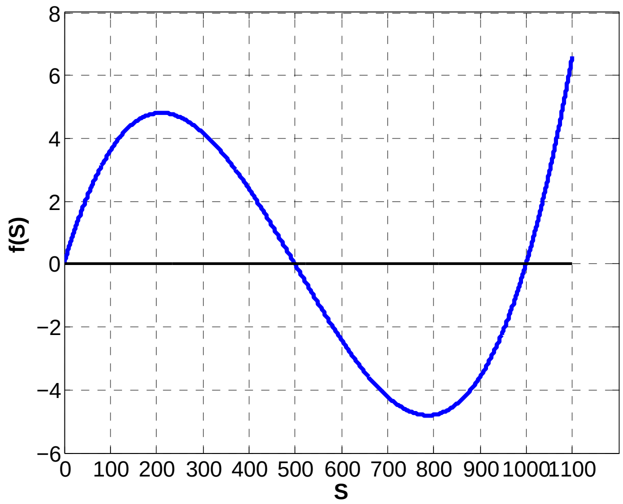

(a) (5 points) The tank contains 20 gallons of water. Initially there are 2 pounds of $Q$ dissolved in the water. Pose a differential equation with initial value for the amount $Q(t)$ in the tank at time $t$.

(b) (6 points) Now suppose that the tank initially contains 2 pounds of $Q$ dissolved in 20 gallons of water. Water containing 1 pound of $Q$ per gallon is added to the tank at the rate of 2 gallons per minute. The volume of water in the tank therefore increases. The filter continues to operate as above at 4 gallons per minute, removing 1/3 of the amount of $Q$ passing through it. Pose a differential equation with initial value for the amount $Q(t)$ in the tank at time $t$.

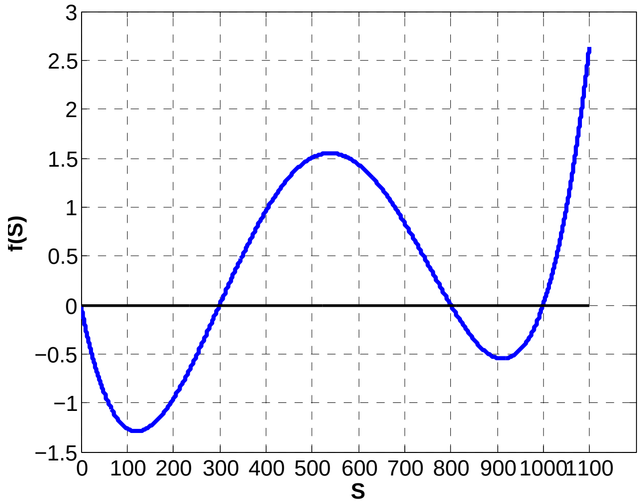
**Problem** 5. Biologists have observed that a population of wild pigs satisfies the following differential equation, where $t$ is in days:

$$\frac{dS}{dt} = f(S).$$

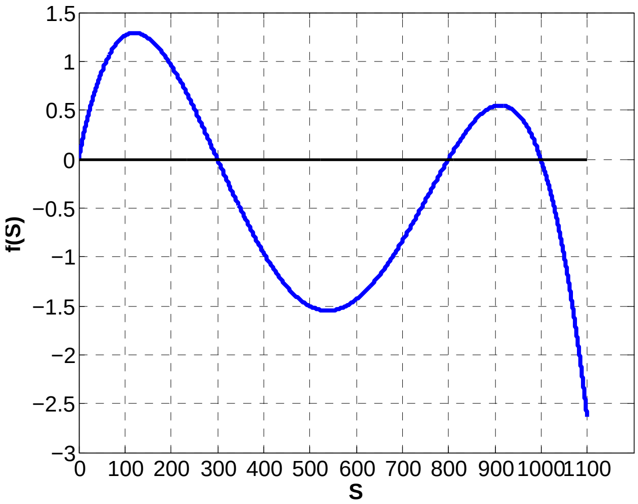
Four proposed expressions for $f(S)$ are shown below:



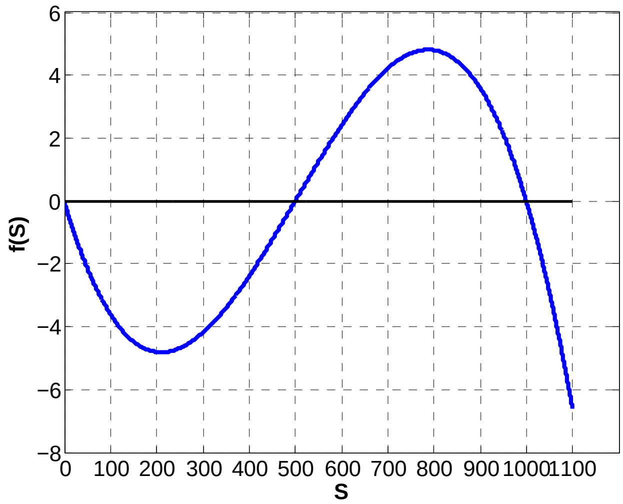
(a) Model A



(b) Model B



(c) Model C



(d) Model D

**Part (a):** (4 points) You observe

- If the population of pigs is 150, the population decreases until it reaches 0.
- If the population of pigs is 600, the population increases until it reaches 1000.

CIRCLE which model illustrates these two observations

Model A Model B Model C Model D

**There are more parts to this problem on the following page!**

**Answer parts (b) and (c) below based on the model you chose in part (a):**

**Part (b):** (8 points) Determine the equilibrium solutions and classify each one as asymptotically stable or unstable. Sketch the direction field, and in your sketch draw an approximate graph of the solution $S(t)$ that satisfies $S(0) = 700$; be sure to indicate the behavior of $S(t)$ as $t$ goes to $+\infty$.

**Part (c):** (4 points) The government plans to hunt a fixed number $k$ of pigs per day. If there are 700 pigs when they start hunting, what value of $k$ (approximately) will keep the population of pigs constant. EXPLAIN.

*Submitted by Name:* ________________________________ *on February 2, 2022.*