Math 307 - Spring 2015
Exam 1
April 22, 2015

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

- There are 5 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators and no calculators that have calculus capabilities**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board. In such an instance, you will meet in front of a board of professors to explain your actions. I have turned in many cases of suspicious work in the past. I will not hesitate to report suspicious work. This is your warning!
  Avoid suspicion of cheating by keeping your eyes on your paper and clearly showing your work on each problem!
- You have 50 minutes to complete the exam. Budget your time wisely.
  SPEND NO MORE THAN 10 MINUTES PER PAGE!

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PAGE 4	10	
PAGE 5	10	
Total	50	

GOOD LUCK!

1. (10 pts)

(a) Find the explicit solution to $\dfrac{dy}{dt} = 5t^4(y-3)^2$ with $y(0) = 4$.

(b) Find the explicit solution to $\dfrac{dy}{dx} = x - \dfrac{3y}{x}$ with $y(1) = \dfrac{21}{5}$. (You may assume $x > 0$).

2. (10 pts)

[redacted]

this problem requires a method we no longer teach in this couse.

(b) Find two different solutions to $\frac{dy}{dt} = 4t\sqrt{y-5}$ with $y(0) = 5$.
AND specifically, and briefly, say why the uniqueness theorem doesn't apply here.
(*i.e.* Tell me what hypothesis of the theorem is false.).

3. (10 pts)

(a) For this question, assume A and B are constants with $A > B > 0$. Find all equilibrium solutions for $\frac{dy}{dt} = (A - y)(y - B)^2(y + 3)$ and classify each as stable, unstable or semistable.

(b) A local pond has an initial volume of 5,000 liters. Water enters the pond through a stream at 400 liters/day and water leaves the pond through a different stream at 600 liters/day. Salt, from the salting of roads, gets into the incoming stream giving it a salt concentration of 0.03 kg/liter. Initially, the pond contains no salt.
Let $y(t)$ is the amount, in kg, of salt in the pond at time t days.
Give the differential equation and initial condition for $y(t)$.
(DO NOT SOLVE, JUST SET UP)

4. (10 pts) A large cylindrical water tank is 10 meters tall and has a small hole in the bottom that is leaking out water. Water is also being pumped in at a constant rate. You are given that

$$\frac{dh}{dt} = 1.5 - 0.5\sqrt{h},$$

where $h = h(t)$ is the height, in meters, of the water in the tank at time t minutes. The tank starts at a height of 4 meters (so $h(0) = 4$ meters).

(a) Approximate the height of the water in the tank in 1 minute using Euler's method with step size 0.5 minutes. In other words, approximate $h(1)$.
(Keep all your work accurate to four digits after the decimal).

(b) Does the tank overflow, completely drain out, or approach a limiting height (if so what is that height)? In other words, find $\lim_{t\to\infty} h(t)$ and explain your work.

5. (10 pts) A person wants to buy a home and they are going to get a home loan. The person can afford to make payments of \$20,000 per year. Assume payments are spread out and paid continuously throughout the year. Also, assume that current interest rates are 5%, compounded continuously.
Just like we did in class and in homework, assume that the rate due to interest is proportional to the balance (with proportionality $r = 0.05$) and that the payments are removed from the balance at a constant rate.
Let $y(t)$ be the loan balance after t years.

   (a) Set up the differential equation AND solve to get $y(t)$.
   Your answer will involve a constant of integration.

   (b) The person plans to get a 30-year loan, which mean the balance will be zero in 30 years. What is the initial loan balance, rounded to the nearest dollar?
   (This is the home loan amount the person can afford).