

Name (PRINT): _____

Student ID (PRINT): _____

**Autumn 2022 – Introduction to Differential Equations
First Examination**

Instructions

1. The use of all electronic devices except a non graphing, non-programmable calculator TI30 or equivalent is prohibited. Any electronic device needs to be turned off and placed in your bag. Any textbooks or notes also need to be placed in your bag.
2. Present your solutions in the space provided. Show all your work neatly and concisely. Clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

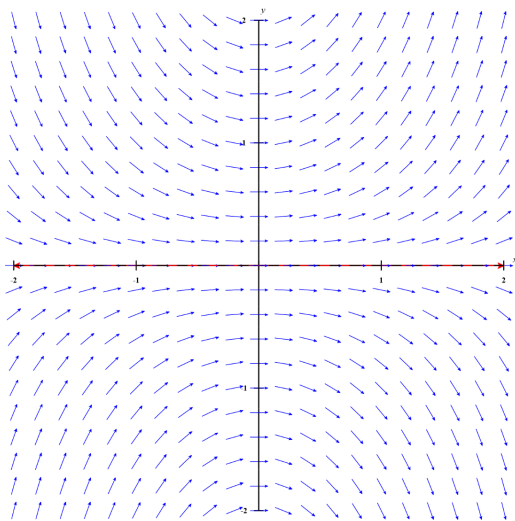
Scholastic dishonesty will not be tolerated and may result in terminating the midterm early. The work on this test is my own.

Signature: _____

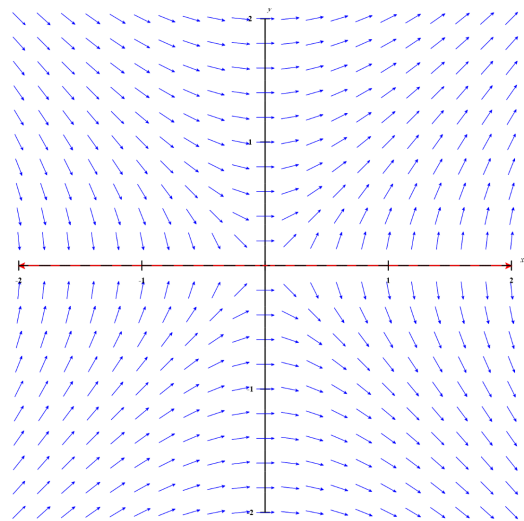
Exercise 1. (4 points) Choose the direction field corresponding to each differential equations. Justify your answers:

1. $y' = \frac{x}{y}$.

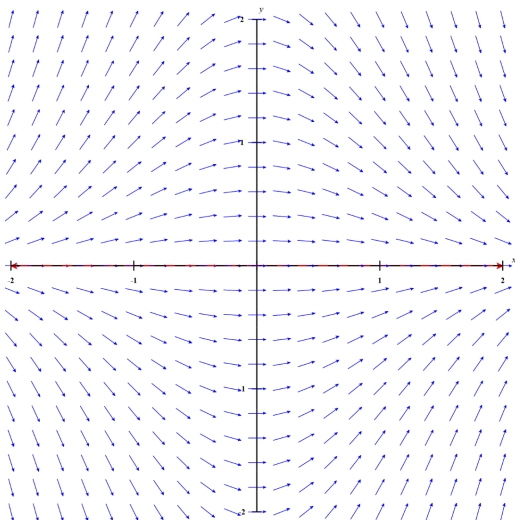
2. $y' = -xy$



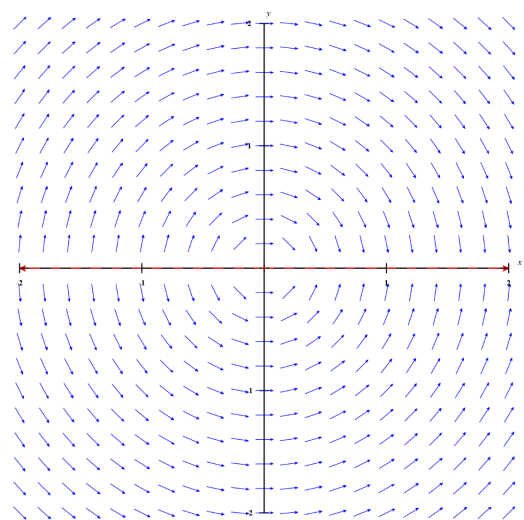
(a) Direction field 1



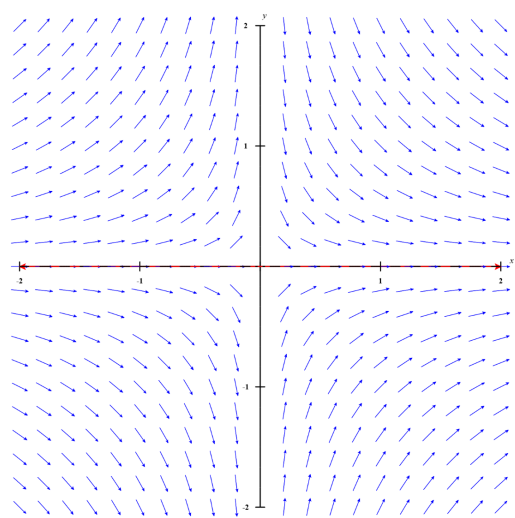
(b) Direction field 2



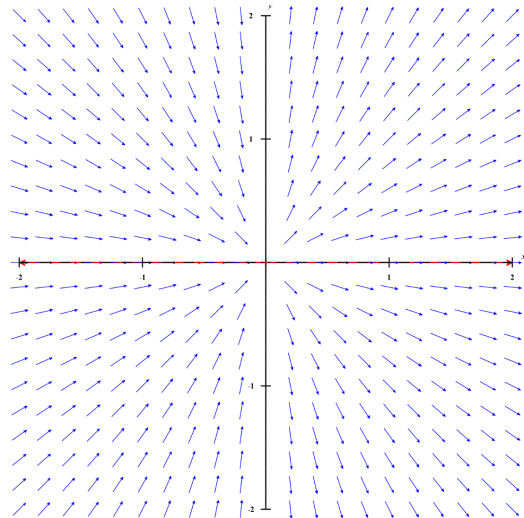
(c) Direction field 3



(d) Direction field 4



(e) Direction field 5



(f) Direction field 6

Exercise 2. (6 points) Find the general solutions. Find explicit solutions if possible.

1.

$$\frac{ty'}{t^4 \cos(2t) + 3y} = 1$$

2.

$$\frac{dy}{dx} = \frac{x^2y^2 + y^2}{x^2y^3 + 4y^2x^2}$$

Exercise 3. (6 points) A tank with capacity of 500gal originally contains 200 gallons of water with 100 lb of salt in solution. Water containing 2 lb of salt per gallon is entering at a rate of 5 gal/min, and the mixture is allowed to flow out of the tank at a rate of 3 gal/min.

1. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow.

2. Find the concentration (in pounds per gallon) of salt in the tank when it is on the point of overflowing.

Exercise 4. (5 points)

1. Solve the initial value problem

$$ty' + 2y = 2\frac{e^{3(t-4)}}{t}, \quad y(4) = 0$$

2. Find $y'(4)$.

Exercise 5. (4 points) Consider the initial value problem

$$y' = 3 + t - y, \quad y(1) = 2$$

1. Use Euler method with step size $1/3$ to get an approximation of $y(0)$.
2. On the interval $[0,1]$, are the solutions concave up or concave down? Justify your answer.