Spring 2018 MATH 307 Final Exam
90 pts total

Name: _____

I, _____ (signature), will not discuss the content of the exam with anyone until noon of Thursday, June 7.

Instruction:

- Nothing but writing utensils and a double side 4in $\times$ 6in notecard are allowed.
- Use the provided Table of Laplace Transforms.
- Unless otherwise specified, you must show work to receive full credit.

1. (13pts) Consider the initial value problem

$$\frac{dy}{dx} = \frac{2x+1}{y-1}, \qquad y(0) = -1.$$

Find $y(1)$.

2. (15pts) Newton's law of cooling states that the rate of change of temperature of an object in a surrounding medium is proportional to the difference of the temperature of the medium and the temperature of the object.

Suppose a metal bar, initially at temperature $T(0) = 40$ degrees Celsius, is placed in a room which is held at the constant temperature of 20 degrees Celsius. One minute later the bar has cooled to 30 degrees. Find $T(t)$ for all time $t > 0$.

Hint: first write the differential equation that models the temperature (in degrees Celsius) as a function of time (in minutes). Start by calling the constant of proportionality k. Solving the initial value problem to obtain the temperature as a function of k and t. Then use the observed temperature after one minute to solve for k.

3. (13pts) Consider the differential equation

$$y'' - 2y' + 2y = te^t \cos t + e^t + \sin t + 1.$$

Write down the form of a particular solution (i.e. the Ansatz), no need to find the constants.

4. (15pts) You have a spring in a damping media, but you don't know the spring constant nor the damping constant. To find that out, you decide to attach a mass of 1 kg to the spring and plot the motion of this unforced damped spring-mass system. The graph below is a plot of the displacement of the mass at any time t. Write down the differential equation governing its motion.

Note: you should write down actual (estimated) numbers based on what you gather from the graph, not just a symbolic equation.

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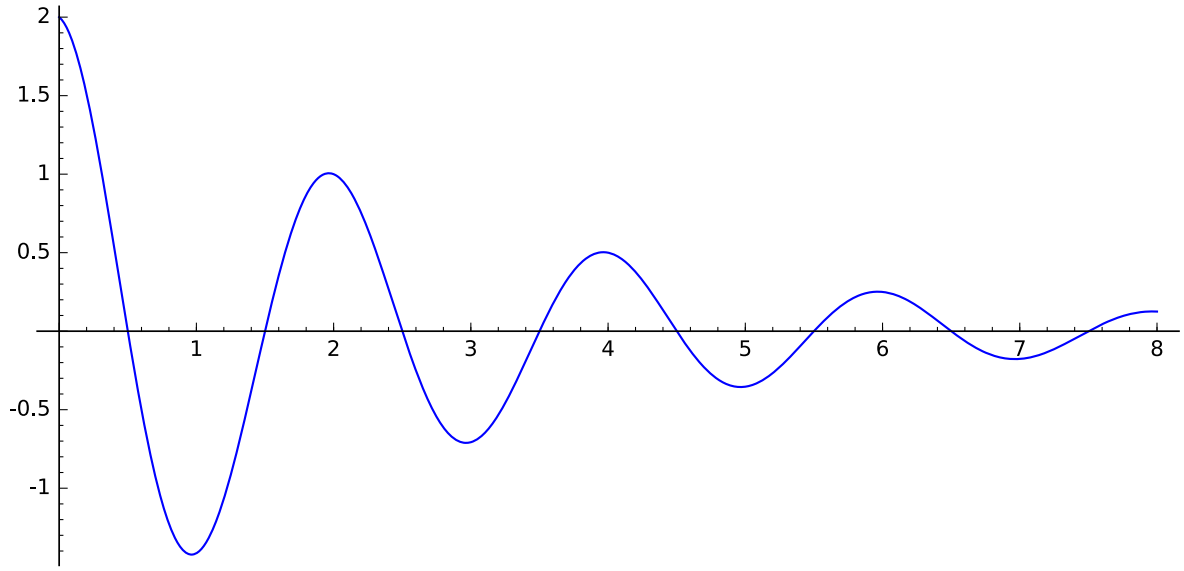

(Extra space in case you need it.)

5. (12pts) Find the Laplace transform

$$\mathcal{L}\{2y''(t) - ty'(t) + 3y(t)\},$$

in terms of $Y(s) = \mathcal{L}\{y\}$ and $Y'(s)$, where $y(0) = 2$ and $y'(0) = -1$.
Hint: use #16 in the table of Laplace transforms.

6. Consider the mechanical vibration modeled by the initial value problem:

$$y''(t) + \pi^2 y(t) = \pi\delta(t - 1.5), \qquad y(0) = -1,\ y'(0) = 0.$$

[a] (14pts) Find the solution to this initial value problem.
Hint: use Laplace transform.

6 (continue)

[**b**] **(8pts)** Graph the solution found in part [a].

Hint: if you couldn't solve part [a], you can still attempt to graph the solution as much as possible to earn partial credits.